Sequences and Series

Arithmetic Sequences

In an arithmetic sequence, consecutive terms change by a constant amount:

u_1	u_2	u_3	...
a	a + d	a + 2d	...

In general the n^{th} term is given by:

$$u_n = a + (n - 1)d$$

Note that d is the **common difference** and **a** is the first term.

Example 1

For the arithmetic sequence 11, 13, 15, 17, …
Find a formula for u_n the n^{th} term, and hence find

a) the 25^{th} term
b) which term is 115
c) which term is first to exceed 200

11, 13, 15, 17,… $a = 11, d = 2$

$$\begin{aligned} u_n &= a + (n - 1)d \\ &= 11 + 2(n - 1) \\ &= 11 + 2n - 2 \\ &= 2n + 9 \end{aligned}$$

a) $u_{25} = 2 \times 25 + 9$
$= 59$

b) $u_n = 115$
$115 = 2n + 9$
$106 = 2n$
$n = 53$

c) $u_n > 200$

$2n + 9 > 200$

$2n > 191$

$n > 95.5$

the 96th term is the first to exceed 200

Example 2

The 4th term of an arithmetic sequence is 31 and the 9th term is 16 Find the 15th term of this sequence.

$u_n = a + (n - 1)d$

$u_4 = 31 \quad \Rightarrow a + 3d = 31 \quad (1)$

$u_9 = 16 \quad \Rightarrow a + 8d = 16 \quad (2)$

(2) – (1) $\Rightarrow 5d = -15$

$d = -3$

Sub in 1 $\Rightarrow a - 9 = 31$

$a = 40$

[Note that $d < 0$ means the terms in the sequence are decreasing]

$u_{15} = a + 14d$

$= 40 + 14 \times -3$

$= -2$

Exercise

1. An arithmetic sequence has first term 6, common difference 3 and $u_n = 72$. Find the value of n.

   n = 23

2. An arithmetic sequence has first term -3, and $u_3 = 14$. Find the value of d.

   d = $\frac{17}{2}$

3. An arithmetic sequence has common difference 9 and $u_{16} = 68$. Find the value of a.

   a = -67