# Algebraic Skills

## The Binomial Theorem

This is the pattern for expanding powers of brackets of the form $( x + y )^n$ directly.

$( x + y )^0 = 1$ We will record this answer as 1

$( x + y )^1 = x + y$ We will record this answer as 11

$( x + y )^2 = ( x + y ) ( x + y )$
$= x^2 + 2xy + y^2$ We will record this answer as 121

$( x + y )^3 = ( x + y ) ( x + y )^2$
$= ( x + y ) ( x^2 + 2xy + y^2 )$
$= x^3 + 2x^2y + xy^2 + yx^2 + 2xy^2 + y^3$
$= x^3 + 3x^2y + 3xy^2 + y^3$ We will record this answer as 1331

$( x + y )^4 = ( x + y ) ( x + y )^3$
$= ( x + y ) ( x^3 + 3x^2y + 3xy^2 + y^3 )$
$= x^4 + 3x^3y + 3x^2y^2 + xy^3 + x^3y + 3x^2y^2 + 3xy^3 + y^4$
$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
We will record this answer as 14641

Note: In the expansion $( x + y )^n$

(1) The sum of the powers of $x$ and y in each term is n

(2) As the powers of $x$ decrease, the powers of y increase

## Pascal's Triangle

The coefficients in the previous expansions can be recorded in a triangular array as follows

| | |
|---|---|
| row 0 (n = 0) | 1 |
| row 1 (n = 1) | 1 1 |
| row 2 | 1 2 1 |
| row 3 | 1 3 3 1 |
| row 4 | 1 4 6 4 1 |
| row 5 | 1 5 10 10 5 1 |
| row 6 | 1 6 15 20 15 6 1 |
| row 7 (n = 7) | 1 7 21 35 35 21 7 1 |

Each row starts and ends with 1 and each number in-between is the sum of the two adjacent numbers in the row above.

The coefficients in every row are also symmetrical.

## Examples
**Level C**

Ex 1. $(x + 5)^5 = 1.x^5 + 5.x^4.5 + 10.x^3.5^2 + 10.x^2.5^3 + 5.x.5^4 + 1.5^5$

(1 5 10 10 5 1) $= \underline{\underline{x^5 + 25x^4 + 250x^3 + 1250x^2 + 3125x + 3125}}$

Ex 2. $(2a - 3)^4 = 1.(2a)^4 + 4.(2a)^3.(-3) + 6.(2a)^2.(-3)^2 + 4.(2a).(-3)^3 + 1.(-3)^4$

(1 4 6 4 1) $= \underline{\underline{16a^4 - 96a^3 + 216a^2 - 216a + 81}}$

Binomial Worksheet Q1 - Q5

## Examples
**Level A/B**

Ex 3. $(2x^2+3)^3 = 1\cdot(2x^2)^3 + 3\cdot(2x^2)^2\cdot 3^1 + 3\cdot(2x^2)^1\cdot 3^2 + 1\cdot(3)^3$

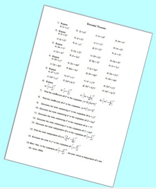
(1 3 3 1) $= \underline{\underline{8x^6 + 36x^4 + 54x^2 + 27}}$

Ex 4.

$$\left(x+\frac{1}{x}\right)^5 = x^5 + 5\cdot x^4\cdot\left(\frac{1}{x}\right)^1 + 10\cdot x^3\cdot\left(\frac{1}{x}\right)^2 + 10\cdot x^2\cdot\left(\frac{1}{x}\right)^3 + 5\cdot x^1\cdot\left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^5$$

$$= \underline{\underline{x^5 + 5x^3 + 10x + \frac{10}{x} + \frac{5}{x^3} + \frac{1}{x^5}}}$$

1,5,10,10,5,1



Binomial Worksheet Q1 - Q5

Ex 5. Find the coefficient of $x^3$ in the expansion $(2x-3)(x+2)^5$

$$(x+2)^5 = x^5 + 5\cdot x^4\cdot 2 + 10\cdot x^3\cdot 2^2 + 10\cdot x^2\cdot 2^3 + 5\cdot x\cdot 2^4 + 2^5$$

$$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

$$(2x-3)(x+2)^5 = (2x-3)(x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32)$$

Term in $x^3 = 2x\cdot 80x^2 - 3\cdot 40x^3$

$= 160x^3 - 120x^3$

$= 40x^3$

Hence the coefficient of $x^3$ is $\underline{\underline{40}}$

**Note**

It is not necessary to expand $(2x-3)(x+2)^5$ fully to obtain the coefficient of one particular term.

Binomial Worksheet Q6 - Q8