Integration

Standard Integrals

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C \qquad (n \neq -1)$$

eg. $\int x^4 dx = \frac{1}{5} x^5 + C$

$$\int (ax+b)^n dx = \frac{1}{a} \cdot \frac{1}{n+1} (ax+b)^{n+1} + C$$

eg $\int (3x+2)^4 dx = \frac{(3x+2)^5}{5 \cdot 3} + C$

$$= \frac{1}{15} (3x+2)^5 + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

eg. $\int \sin(3x-2)\,dx = -\frac{1}{3}\cos(3x-2) + C$

$$\int \cos x\,dx = \sin x + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

eg. $\int \cos(2x+1) = \frac{1}{2}\sin(2x+1) + C$

Reminders

$$\frac{4}{x^3} = 4x^{-3}$$

$$\sqrt{x} = x^{1/2}$$

$$\sqrt[4]{x^3} = x^{3/4}$$

$$\frac{x^2 - x}{\sqrt{x}} = \frac{x^2}{x^{1/2}} - \frac{x}{x^{1/2}}$$

$$= x^{3/2} - x^{1/2}$$

Definite Integrals

$$\int_1^2 \left(x^2 + x + 1\right)dx = \left[\frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right]_1^2$$

$$= \left[\frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 + 2\right] - \left[\frac{1}{3}(1)^3 + \frac{1}{2}(1)^2 + 1\right]$$

$$= \left[\frac{8}{3} + \frac{4}{2} + 2\right] - \left[\frac{1}{3} + \frac{1}{2} + 1\right]$$

$$= 6\frac{2}{3} - 1\frac{5}{6} = 4\frac{5}{6}$$

Examples

1. $\int \frac{3x^2 - 1}{\sqrt{x}}dx$ $\qquad \frac{6}{5}x^{\frac{5}{2}} - 2\sqrt{x} + C$

2. $\int (2x+3)^5 dx$ $\qquad \frac{1}{12}(2x+3)^6 + C$

3. $\int \frac{2}{(3x+1)^5}dx$ $\qquad -\frac{1}{6(3x+1)^4} + C$

4. $\int (3\cos x - 4\sin 2x)\,dx$ $\qquad 3\sin x + 2\cos 2x + C$

5. $\int_0^2 \sqrt{(4x+1)}dx$ $\qquad 4\frac{1}{3}$