Vectors

Intersection of a Line and a Plane

Example

Find the point of intersection of the line $\frac{x-7}{3} = \frac{y-11}{4} = \frac{z-24}{13}$

and the plane $6x + 4y - 5z = 28$

Line : $x = 3t + 7,\quad y = 4t + 11,\quad z = 13t + 24$

Subst. into equation of Plane :

$$6(3t + 7) + 4(4t + 11) - 5(13t + 24) = 28$$
$$18t + 42 + 16t + 44 - 65t - 120 = 28$$
$$-31t - 34 = 28$$
$$t = -2$$

$\Rightarrow$ point of intersection has parameter $t = -2$

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Line : $x = 3t + 7,\quad y = 4t + 11,\quad z = 13t + 24$

Point of Intersection :

$x = 3(-2) + 7,\quad y = 4(-2) + 11,\quad z = 13(-2) + 24$

$= 1 \qquad = 3 \qquad = -2$

$= (1, 3, -2)$

Example

Show that, for all values of a except one, the line given by:

$$x = a + \lambda,\quad y = 2 + \lambda,\quad z = 1 - \lambda$$

does not intersect the plane with the equation:

$$2x - y + z = 1$$

$x = a + \lambda, \quad y = 2 + \lambda, \quad z = 1 - \lambda$ $\qquad\qquad$ $2x - y + z = 1$

Substituting the parametric equations into the equation of the plane gives:

$$2(a + \lambda) - (2 + \lambda) + (1 - \lambda) = 1$$
$$2a - 1 = 1$$
$$a = 1$$

So, for all a $\neq 1$ we are left with an impossibility.

Hence for all values of a except 1, the line does not intersect the plane (they are parallel).

ex10 page 68 Q1(i)def, 5, 6