Vectors

Intersection of Three Planes

Three planes may:

(1) intersect at a point
(2) intersect in a line
(3) not all intersect (only intersect two at a time)

Since each plane has an equation of the form

$$ax + by + cz + d = 0,$$

the intersection is a solution of a 3×3 system of equations.

Example (a point of intersection)

$$x + 3y + 2z = 8$$
$$x + 4y + z = 13$$
$$2x + 9y - 3z = 35$$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 1 & 4 & 1 & 13 \\ 2 & 9 & -3 & 35 \end{array}\right) \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_3 \rightarrow R_3 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 1 & 4 & 1 & 13 \end{array}\right)$$

$R_2 \rightarrow R_2 - R_1$

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \end{array}\right)$$

From R_3

From R_2

From R_1

Pt. of Intersection
(2, -1, 4)

[Matrix Revision: Redundancy]

Example (a line of intersection)

Solve the system of equations:

$$x + y + z = 6$$
$$2x + y - 2z = -2$$
$$3x + 2y - z = 4$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & -2 & -2 \\ 3 & 2 & -1 & 4 \end{array}\right) \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ & & & \\ & & & \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ & & & \\ & & & \end{array}\right)$$

The row of zeros gives no information
⇒ infinite number of solutions

Let $z = t$, to give the parametric equations

From R_2: $-y - 4z = -14 \quad (z = t)$

$y = 14 - 4t$

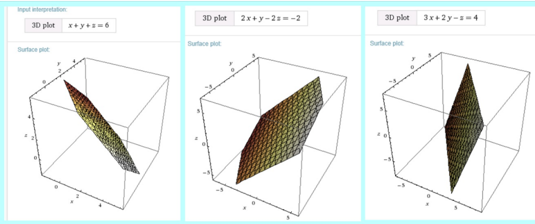
From R_1: $x + y + z = 6 \qquad (y = 14 - 4t \text{ and } z = t)$

$x + (14 - 4t) + t = 6$

$x = 3t - 8$

Hence solution can be written $x = 3t - 8$, $y = 14 - 4t$, $z = t$

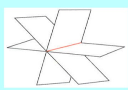
or in Symmetric Form : $\dfrac{x+8}{3} = \dfrac{y-14}{-4} = \dfrac{z}{1}$



These equations give points that all lie on a straight line.

The system of equations has an infinite number of solutions.

There are, really, only two equations, & the 3rd is **redundant**.



[Matrix Revision : Inconsistent]

Example (no intersection)

Solve the system of equations:
$$x + y + z = 6$$
$$2x + y - 2z = -2$$
$$3x + 2y - z = 3$$

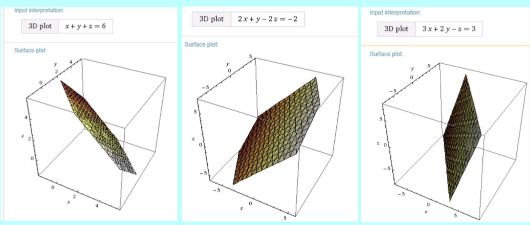
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & -2 & -2 \\ 3 & 2 & -1 & 3 \end{array}\right) \quad \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ & & & -14 \\ & & & -15 \end{array}\right)$$

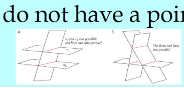
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ & & & -14 \\ & & & -1 \end{array}\right)$$

From R_3 we get $0z = -1$ (inconsistent) ⇒ no solutions

All three planes do not intersect.



Inconsistency means that there are no values of x, y, z which satisfy all three equations. It arises when three planes, given by the 3 equations, do not have a point in common.



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2009 Q16a 5 marks