Complex Numbers

Complex Conjugate

The **complex conjugate** of $z = x + yi$ is

$$\bar{z} = x - yi$$

$w = 1 + 2i$	$\bar{w} = 1 - 2i$
$z = 5 - 3i$	$\bar{z} = 5 + 3i$
$w = 4i$	$\bar{w} = -4i$
$z = 3$	$\bar{z} = 3$

Complex Conjugate

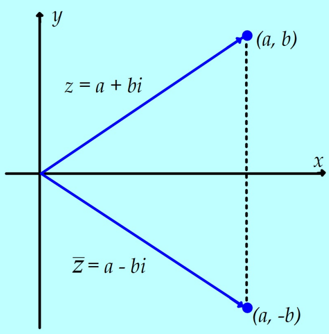
The **complex conjugate** of $z = x + yi$ is

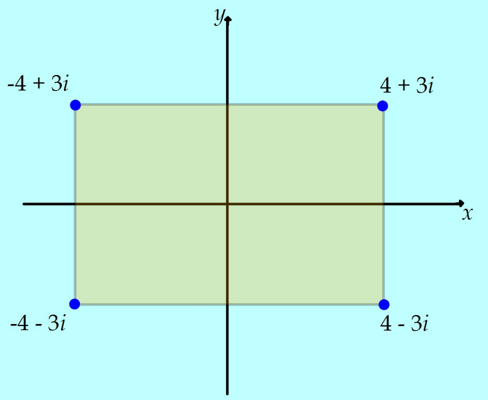
$$\bar{z} = x - yi$$

$$z\bar{z} = (a + bi)(a - bi)$$

The product of a pair of complex numbers is a real number.

Geometrically, $\bar{z}$ is the reflection of z about the real axis.





Operations on Complex Numbers

Example

Let $z = 4 + 3i \qquad w = 1 - 2i$

DIVISION

$$\frac{z}{w} = \frac{4+3i}{1-2i}$$

$$= \frac{4+3i}{1-2i} \times \frac{1+2i}{1+2i}^{*}$$

$$= \frac{4+11i+6i^2}{1^2-4i^2}$$

$$= \frac{4+11i-6}{5}$$

$$= \frac{-2+11i}{5} \qquad = -\frac{2}{5} + \frac{11}{5}i$$

* $\overline{w} = 1 + 2i$ is the **complex conjugate** of w; we rationalise the denominator

Example 2

Find the square roots of $3 + 4i$.

Let $\sqrt{3+4i} = x + iy$

$$(x+iy)^2 = 3 + 4i$$

$$x^2 + i^2y^2 + 2xyi = 3 + 4i$$

Equating **Real** and **Imaginary** parts $\qquad i^2 = -1$

$$x^2 - y^2 = 3$$

$$2xy = 4 \qquad y = \tfrac{2}{x}$$

Subst $y = \tfrac{2}{x}$ into $x^2 - y^2 = 3$

Subst $y = \tfrac{2}{x}$ into $x^2 - y^2 = 3$

$$x^2 - \left(\tfrac{2}{x}\right)^2 = 3$$

$$x^4 - 4 = 3x^2$$

$$x^4 - 3x^2 - 4 = 0$$

$$\left(x^2-4\right)\left(x^2+1\right) = 0$$

$x^2 - 4 = 0 \qquad\qquad x^2 + 1 = 0$

$x^2 = 4 \qquad\qquad x^2 = -1$ **NO SOLUTION SINCE x IS REAL**

$x = \pm 2$

Using $y = \tfrac{2}{x}$

When $x = 2$, $y = 1$

When $x = -2$, $y = -1$

BE CAREFUL ($x + yi$)

Solutions are $2 + i$ and $-2 - i$