Type 2: Q(x) the denominator has a repeated linear factor.

Example 1

Express $\frac{3x^2-6x-13}{(x+3)(x-1)^2}$ in partial fractions.

$$\frac{3x^2-6x-13}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

× both sides by $(x+3)(x-1)^2$

$$3x^2-6x-13 = A(x-1)^2 + B(x+3)(x-1) + C(x+3)$$

Let $x=1$ $\quad C = -4$

Let $x=-3$ $\quad A=2$

Let $x=0$ $\quad -13 = A - 3B + 3C$

$B = 1$

$$\frac{3x^2-6x-13}{(x+3)(x-1)^2} = \frac{2}{(x+3)} + \frac{1}{(x-1)} - \frac{4}{(x-1)^2}$$

Example 2

Express $\frac{-8x^2+14x-15}{4x^3+4x^2-7x+2}$ in partial fractions.

We have to factorise the denominator by synthetic division.

-2	4	4	-7	2
		-8	8	-2
	4	-4	1	0

$(4x^2 - 4x + 1)(x + 2)$

$= (2x - 1)^2(x + 2)$

$$\frac{-8x^2+14x-15}{4x^3+4x^2-7x+2} = \frac{A}{2x-1} + \frac{B}{(2x-1)^2} + \frac{C}{x+2}$$

× both sides by $(2x - 1)^2(x + 2)$

$$-8x^2 + 14x - 15 = A(2x - 1)(x + 3) + B(x + 2) + C(2x - 1)^2$$

$$-8x^2 + 14x - 15 = A\left(2x - 1\right)\left(x + 3\right) + B(x + 2) + C(2x - 1)^2$$

Let $x = -2$ $\quad C = -3$

Let $x = \frac{1}{2}$ $\quad B = -4$

Let $x = 0$ $\quad A = 2$

$$\frac{-8x^2+14x-15}{4x^3+4x^2-7x+2} = \frac{2}{2x-1} - \frac{4}{(2x-1)^2} - \frac{3}{x+2}$$