Integration

Integration of Exponential Functions

The most special exponential function is e^x.

Since $\frac{d}{dx}(e^x) = e^x$ i.e. it is its own derivative

Hence

$$\int e^x dx = e^x + C$$

$$\int e^{(ax+b)} dx = \frac{1}{a} e^{(ax+b)} + C$$

Note that the second standard integral only applies when integrating the exponential of a linear function. It cannot be used to find integrals such as $\int e^{x^2} dx$.

Formula Sheet

Standard derivatives	
$f(x)$	$f'(x)$
$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1}x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\ln x,\ x>0$	$\frac{1}{x}$
e^x	e^x

Standard integrals	
$f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Ex 1

$$\int (6e^{3x}+4e^{-x})dx = 6.\frac{1}{3}e^{3x}+4.\frac{1}{(-1)}e^{-x}+C$$

$$= \underline{\underline{2e^{3x}-4e^{-x}+C}}$$

Ex 2

$$\int \frac{8}{e^{2x}}dx = \int 8e^{-2x}dx$$

$$= 8.\frac{1}{(-2)}e^{-2x}+C$$

$$= \underline{\underline{-4e^{-2x}+C}}$$

Ex 3

Show that $\int \left(e^x + \frac{1}{e^x} \right)^2 dx = \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C$

$$\int \left(e^x + \frac{1}{e^x} \right)^2 dx = \int \left(e^x + \frac{1}{e^x} \right)\left(e^x + \frac{1}{e^x} \right) dx$$

$$= \int \left(e^{2x} + 2 + \frac{1}{e^{2x}} \right) dx$$

$$= \int \left(e^{2x} + 2 + e^{-2x} \right) dx$$

$$= \frac{1}{2}e^{2x} + 2x + \frac{1}{(-2)}e^{-2x} + C$$

$$= \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C$$

Ex 4

Find the exact value of the definite integral $\int_0^1 \frac{\left(e^{2x} - 1\right)^2}{e^x} dx$

$$\int_0^1 \frac{\left(e^{2x}-1\right)^2}{e^x}dx = \int_0^1 \frac{e^{4x}-2e^{2x}+1}{e^x}dx$$

$$= \int_0^1 (e^{3x}-2e^x+e^{-x})dx$$

$$= \left[\frac{1}{3}e^{3x} - 2e^x - e^{-x}\right]_0^1$$

$$= \left[\frac{1}{3}e^3 - 2e^1 - e^{-1}\right] - \left[\frac{1}{3}e^0 - 2e^0 - e^0\right]$$

$$= \left(\frac{1}{3}e^3 - 2e - e^{-1}\right) - \left(\frac{1}{3} - 2 - 1\right)$$

$$= \underline{\underline{\frac{1}{3}e^3 - 2e - \frac{1}{e} + \frac{8}{3}}}$$