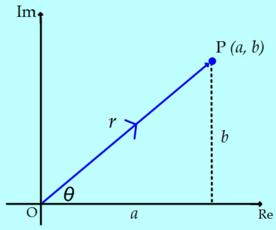
Complex Numbers

Argand Diagrams - Modulus and Argument

A diagram showing complex numbers is called an ARGAND DIAGRAM.



$z = a + bi$ is represented by

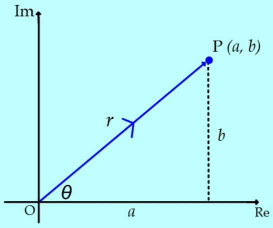
(1) the point P(a, b)

or (2) the vector $\overrightarrow{OP}$

Modulus of *z*

$|z|$ = length of $\overrightarrow{OP}$

$|z| = r = \sqrt{a^2 + b^2}$



Example

Find $|z|$ if $z = 3 - 4i$.

$a = 3$, $b = -4$

$$|z| = \sqrt{a^2 + b^2}$$

$$|z| = \sqrt{3^2 + (-4)^2}$$

$$|z| = \sqrt{25}$$

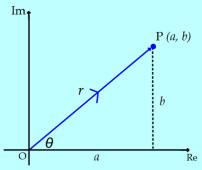
$$|z| = 5$$

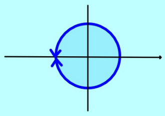
Argument of z

$$Arg(z) = \theta$$

This is the angle between the real axis and OP such that $-180 < \theta < 180$

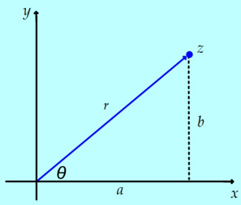
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$





Calculations for θ depend on the quadrant z lies in.

Polar Form of a Complex Number



$cos\theta = \frac{a}{r} \Rightarrow a = rcos\theta$

$sin\theta = \frac{b}{r} \Rightarrow b = rsin\theta$

$\therefore \quad z = a + bi$

$z = rcos\theta + rsin\theta$

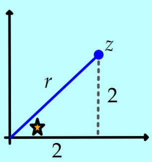
$z = r(cos\theta + sin\theta)$

$$z = r(cos\theta + sin\theta)$$

Examples

Quadrant 1

Express $z = 2 + 2i$ in polar form.



$r^2 = 2^2 + 2^2$

$r = \sqrt{8}$

$r = 2\sqrt{2}$

$\tan(\theta) = \frac{2}{2}$

$\theta = \tan^{-1}(1)$

$\theta = 45°$

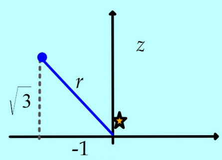
$z = r(\cos\theta + i\sin\theta)$

$z = 2\sqrt{2}(\cos 45 + i\sin 45)$

Examples

Quadrant 2

Express $z = -1 + \sqrt{3}i$ in polar form.



$r^2 = (-1)^2 + (\sqrt{3})^2$

$r = \sqrt{4}$

$r = 2$

$\theta = 180° - \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\theta = 120°$

2nd Quadrant

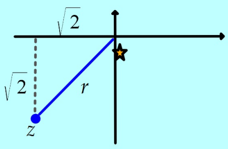
$z = r(\cos\theta + i\sin\theta)$

$z = 2(\cos 120 + i\sin 120)$

Examples

Quadrant 3

Express $z = -\sqrt{2} - \sqrt{2}i$ in polar form.



$$r^2 = \left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2 \qquad \tan(\theta) = -\frac{\sqrt{2}}{\sqrt{2}}$$

$$r = \sqrt{4}$$

$$r = 2$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45°$$

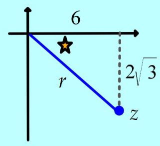
$$z = r(\cos\theta + i\sin\theta)$$

$$\underline{z = 2(\cos(-135) + i\sin(-135))}$$

Examples

Quadrant 4

Express $z = 6 - 2\sqrt{3}i$ in polar form.



$$r^2 = 6^2 + \left(2\sqrt{3}\right)^2 \qquad \tan(\theta) = -\frac{2\sqrt{3}}{6}$$

$$r = \sqrt{48}$$

$$r = 4\sqrt{3}$$

$$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{6}\right)$$

$$\theta = 30°$$

$$z = r(\cos\theta + i\sin\theta)$$

$$\underline{z = 4\sqrt{3}\left(\cos(-30) + i\sin(-30)\right)}$$

Examples

Quotient Form

Express $\frac{1+3i}{1-2i}$ in the form x + iy.

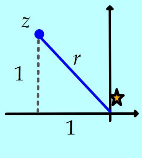
Hence express in polar form.

$$\frac{1+3i}{1-2i} = \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{1+5i+6i^2}{1-4i^2}$$

$$= \frac{-5+5i}{5}$$

$$= -1 + i$$



$$\frac{1+3i}{1-2i} = -1 + i$$

$$r^2 = 1^2 + 1^2 \qquad \tan(\theta) = -\frac{1}{1}$$

$$r = \sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{1}\right)$$

$$\theta = 135°$$

$$z = r(\cos\theta + i\sin\theta)$$

$$\underline{z = \sqrt{2}\left(\cos 135 + i\sin 135\right)}$$