# Sequences and Series

Arithmetic Sequences

## Example 1

Find the sum of the first 15 terms of the arithmetic series
3 + 8 + 13 + 18 + ...

a = 3 and d = 5

$$s_n = \frac{n}{2}(2a + (n-1)d)$$

$$s_{15} = \frac{15}{2}(2(3) + (14)5)$$

$$= 7.5 \times 76$$

$$= 570$$

## Example 2

Find the sum of the series 6 + 7.6 + 9.2 + 10.8 + ... + 49.2

$a = 6, d = 1.6$

We must determine how many terms are in the series

$u_n = 49.2$

$$a + (n-1)d = 49.2$$

$$6 + 1.6(n-1) = 49.2$$

$$6 + 1.6n - 1.6 = 49.2$$

$$1.6n = 44.8$$

$$n = 28$$

There are 28 terms in the series

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{28} = \frac{28}{2}(2(6) + 27(1.6))$$

$$= \underline{\underline{772.8}}$$

## Example 3

For the arithmetic sequence 2, 5, 8, 11, ... find the sum of the 15th to the 35th term (both terms inclusive).

$a = 2, d = 3 \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$$S_{35} = \frac{35}{2}\left(2(2) + 34(3)\right) = 1855$$

$$S_{14} = \frac{14}{2}\left(2(2) + 13(3)\right) = 301$$

$$S_{35} - S_{14} = 1855 - 301 = \underline{\underline{1554}}$$

## Example 4

Find a formula for $S_n$, the sum of the first $n$ terms of the sequence 9, 15, 21, 27, ...

Hence find the least number of terms which must be taken to give a sum excededing 10 000.

$a = 9, d = 6$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$
$$= \frac{n}{2}\left(18 + 6(n-1)\right)$$
$$= \frac{n}{2}(6n + 12)$$
$$= 3n^2 + 6n$$

$S_n = 10\ 000$

$10\ 000 = 3n^2 + 6n$

$3n^2 + 6n - 10\ 000 = 0$

$a = 3, b = 6, c = -10\ 000$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad b^2 - 4ac = 6^2 - 4 \times 3 \times (-10\ 000) = 120\ 036$$

$$n = \frac{-6 \pm \sqrt{120\,036}}{6}$$

$n = 56.7$ or $n = -58.7$

$n = 56.7$ is the only valid solution since $n$ must be positive.
We need 57 terms until the sum exceeds 10,000.

## Example 5

Let $u_n$ denote the $n^{th}$ term of the arithmetic sequence 2, 10, 18, 26.

Let $S_n = \sum_{k=1}^{n} u_k$

(a) Find a formula for $S_n$ in terms of $n$.
(b) Find the least value of n for which $S_n > 1000$.
(c) Evaluate $\sum_{k=20}^{40} u_k$

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Let $S_n = \sum_{k=1}^{n} u_k$

(a) Find a formula for $S_n$ in terms of $n$.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$= \frac{n}{2}\big(4 + 8(n-1)\big)$$
$$= \frac{n}{2}(8n - 4)$$
$$= 4n^2 - 2n$$

(b) Find the least value of n for which $S_n > 1000$.

$$4n^2 - 2n = 1000$$
$$2n^2 - n - 500 = 0$$
$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{1 \pm \sqrt{4001}}{4}$$

$n = 16.06 \quad or \quad n = -15.56$

But $n > 0$, so $n = 16.06$.

So it is the 17th term.

(c) Evaluate $\sum_{k=20}^{40} u_k$

$$\sum_{k=20}^{40} u_{20} + u_{21} + u_{22} + \cdots + u_{40}$$
$$= (u_1 + u_2 + \cdots + u_{40}) - (u_1 + u_2 + \cdots + u_{19})$$
$$= S_{40} - S_{19}$$
$$= 4914$$

## Example 6

The terms of a sequence are given by $u_k = 11 - 2k, k \geq 1$

(a) Obtain a formula for $S_n$, where $S_n = \sum_{k=1}^{n} u_k$

(b) Find the values of n for which $S_n = 21$.

$u_k = 11 - 2k$

$u_1 = 9 \quad u_2 = 7 \quad u_3 = 5$


$$a = 9, d = -2 \quad S_n = \frac{n}{2}(2a + (n-1)d)$$
$$= 10n - n^2$$

(b)

$$10n - n^2 = 21$$

$$n^2 - 10n + 21 = 0$$

$$(n-7)(n-3) = 0$$

$$n = 7 \quad or \quad n = 3$$

## Example 7

An arithmetic sequence has first term 12 and $S_{14}$ = 238.
Find the common difference.

$$d = \frac{10}{13}$$