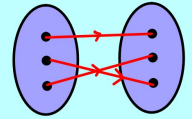


# Functions

## Inverse Functions

When function  $f$  is a one-to-one correspondence from  $A$  to  $B$ , there exists an INVERSE function,  $f^{-1}$ , that maps  $B$  to  $A$ .



$$y = f(x) \Rightarrow x = f^{-1}(y)$$

$$f^{-1}(f(x)) = x = f(f^{-1}(x))$$

i.e. for any value  $y$  there is a unique value of  $x$

### Example

$$f(x) = 4x^2 + 1$$

$$y = 4x^2 + 1$$

$$4x^2 = y - 1$$

$$x^2 = \frac{y-1}{4}$$

$$x = \frac{\sqrt{y-1}}{2} \quad \text{(taking positive root)}$$

So  $f^{-1}(x) = \frac{\sqrt{x-1}}{2}$

**\*\* The Domain and Range of  $f^{-1}$  give the Range and Domain of  $f$  \*\***

Function	Domain	Range	Graph
$f(x) = 4x^2 + 1$	$x \geq 0, x \in \mathbb{R}$	$y \geq 1, y \in \mathbb{R}$	
$f^{-1}(x) = \frac{\sqrt{x-1}}{2}$	$x \geq 1, x \in \mathbb{R}$	$y \geq 0, y \in \mathbb{R}$	

#### Note

The graph of  $y = f^{-1}(x)$  is the reflection of the graph of  $y = f(x)$  in the line  $y = x$