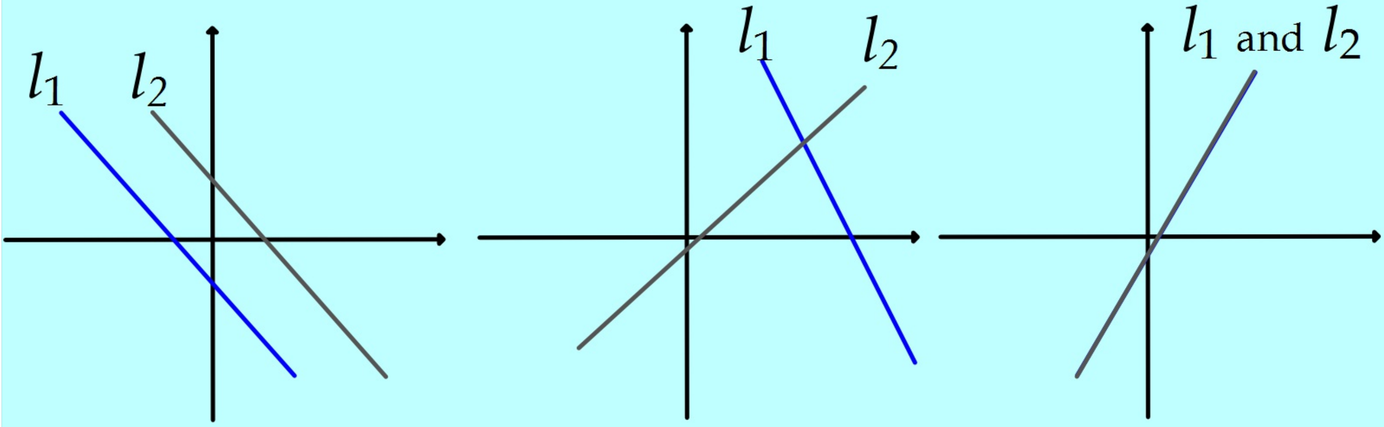
# Redundant, Inconsistent & Ill-conditioned Equations

"Explain what happens when ..............."

# Linear Systems

We have considered only two equations with two unknowns in the past, and will now show that the same three possibilities hold for arbitrary linear systems:




*(a)* No solution    *(b)* One solution    *(c)* Infinitely many solutions

*Every system of linear equations has no solutions, or has exactly one solution, or has infinitely many solutions.*

# Redundancy

**Example**

Solve the system of equations:

$$x + y + z = 6$$
$$2x + y - 2z = -2$$
$$3x + 2y - z = 4$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & -2 & -2 \\ 3 & 2 & -1 & 4 \end{array}\right) \quad \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$$\begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & -1 & -4 & -14 \end{array}\right)$$

$$R_3 \to R_3 - R_2 \quad \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

The row of zeros gives no information and tells us that there is not a unique solution for $z$. (i.e. $z$ can take <u>any</u> value).

Let $z = t$ (where $t$ is any real number)

From $R_2$: $-y - 4z = -14 \quad (z = t)$

$$y = 14 - 4t$$

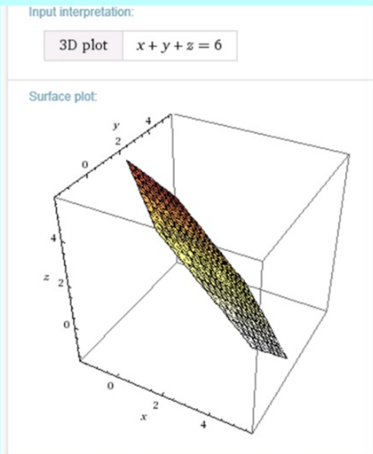
From $R_1$: $x + y + z = 6 \quad (y = 14 - 4t \text{ and } z = t)$

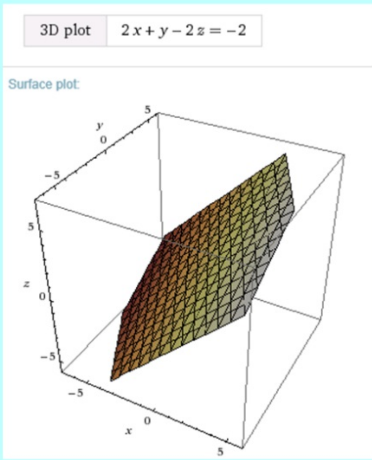
$$x + (14 - 4t) + t = 6$$

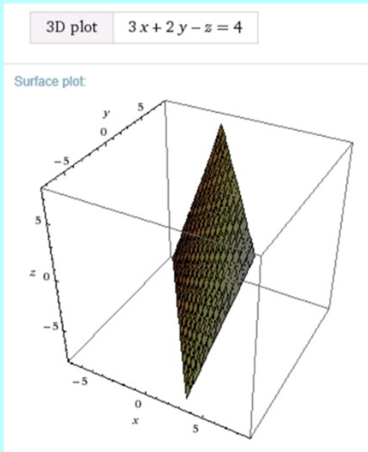
$$x = 3t - 8$$

Hence solution can be written $x = 3t - 8$, $y = 14 - 4t$, $z = t$

A general solution as shown above is usually given for redundancy.



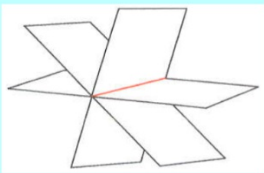




These equations give points that all lie on a straight line *(see Vectors Unit 3)*.

The system of equations has an infinite number of solutions.

There are, really, only two equations, & the 3rd is **redundant**.



# Inconsistent

**Example**

Solve the system of equations:

$$x + y + z = 6$$
$$2x + y - 2z = -2$$
$$3x + 2y - z = 3$$

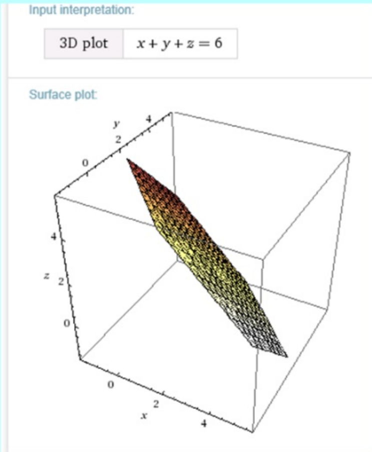
$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & -2 & -2 \\ 3 & 2 & -1 & 3 \end{array}\right) \quad \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

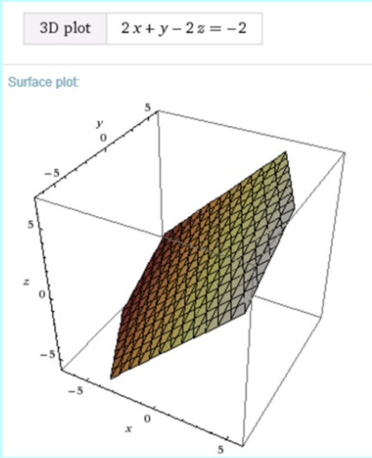
$$\begin{array}{c} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & -1 & -4 & -15 \end{array}\right)$$

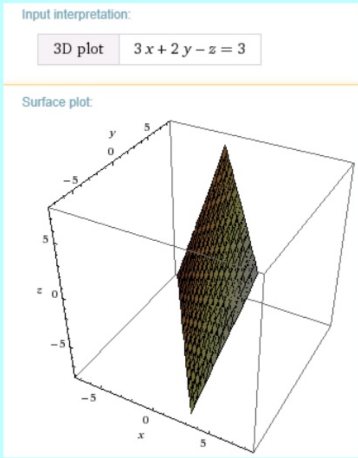
$$\begin{array}{c} \\ \\ R_3 \to R_3 - R_2 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -4 & -14 \\ 0 & 0 & 0 & -1 \end{array}\right)$$

From $R_3$ we get $0z$ = -1 which is clearly impossible.

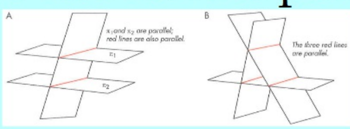
Hence there is no solution.




**Inconsistency** means that there are no values of *x*, *y*, *z* which satisfy all three equations. It arises when three planes, given by the 3 equations, do not have a point in common.



# Ill-conditioning

All our calculations so far have been exact. However, in many real-life problems using experimental or collected data, the coefficients in the equations are often rounded to the nearest integer, 1 d.p., etc.

**Example**

Consider these results from an experiment:

$$\begin{aligned} x + 1000y &= 1 \\ x + 999y &= 2 \end{aligned} \qquad \begin{aligned} x + 999y &= 1 \\ x + 1000y &= 2 \end{aligned}$$

$$x + 1000y = 1$$
$$x + \quad 999y = 2$$

$$\begin{pmatrix} 1 & 1000 & 1 \\ 1 & 999 & 2 \end{pmatrix}$$

$$R_2 - R_1 \quad \begin{pmatrix} 1 & 1000 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$Sol^n \; is \; x = 1001, \;\; y = \; -1$$

$$x + \quad 999y = 1$$
$$x + 1000y = 2$$

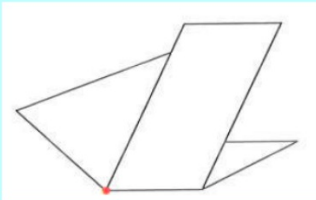
$$\begin{pmatrix} 1 & 999 & 1 \\ 1 & 1000 & 2 \end{pmatrix}$$

$$R_2 - R_1 \quad \begin{pmatrix} 1 & 999 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

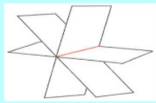
$$Sol^n \; is \; x = \; -998, \;\; y = \; 1$$

Where a system of eq$^n$s show ill-conditioning no confidence can be put on the results obtained and is ∴ not a good model for real-life contexts.

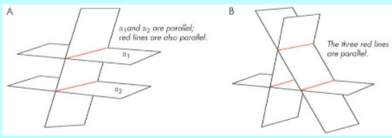
## Summary



**Redundant:**
When the second and third rows are the same i.e. redundant (provides no further information) there is an infinite number of solutions.



**Inconsistent:**
When no solutions exist i.e. $0z = 6$.




**Ill-conditioned:**
Disproportiate effects to results as a result of small changes to coefficients of $x$, $y$ and $z$.