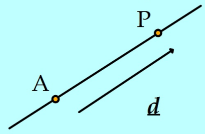
Vectors

Equation of a Line (3 Dimensions)



Vector Form

Let P be any point on the line L.

L passes through the point A with direction $\underline{d}$

$$\overrightarrow{AP} = t\underline{d}$$

$$\underline{p} - \underline{a} = t\underline{d}$$

$$\underline{p} = \underline{a} + t\underline{d} \qquad \text{(vector equation)}$$

Equation of a Line (3 Dimensions)

Parametric Form

P (x, y, z) , A (x_1, y_1, z_1) and $\underline{d} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + t\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$\Rightarrow x = x_1 + ta$$
$$y = y_1 + tb$$
$$z = z_1 + tc$$

(parametric equations)
(t is the parameter)

Equation of a Line (3 Dimensions)

Symmetric Form

(change subject of parametric equations to t)

$x = x_1 + ta$ $\quad y = y_1 + tb$ $\quad z = z_1 + tc$ (parametric equations)

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = t$$

Example

Find the equations of the line passing through A(2, 1, 3) and B(3, 4, 5).

Direction of Line ($\underline{d}$) = $\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

Point on Line = A(2, 1, 3)

Vector Equation: $\underline{p} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

Parametric Equations: $x = 2 + t, \quad y = 1 + 3t, \quad z = 3 + 2t$

Symmetric Form: $\frac{x-2}{1} = \frac{y-1}{3} = \frac{z-3}{2}$

Direction of Line = $\overrightarrow{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

Point on Line = A(2, 1, 3)

Symmetric Form: $\frac{x-2}{1} = \frac{y-1}{3} = \frac{z-3}{2}$

Note (1)/ the coordinates of the point on the line can be read from the numerators and the vector parallel ($\underline{d}$) to the line can be read from the denominators.

Example

Find the symmetric form of the equation of the line through the point (6, 3, -5).

(i) in the direction $\begin{pmatrix} 4 \\ -8 \\ 7 \end{pmatrix}$

(ii) parallel to the line $\frac{x}{3} = \frac{y-10}{-2} = \frac{z+8}{13}$

(i) $\frac{x-6}{4} = \frac{y-3}{-8} = \frac{z+5}{7}$

Numerator: point on line
Denominator: direction

(ii) $\frac{x-6}{3} = \frac{y-3}{-2} = \frac{z+5}{13}$

Since both lines must have the same direction vector.

Note (2)/

(1) "= t" is often omitted in the symmetric form

(2) if any component of $\underline{d}$ is 0 then some parts of the symmetric form will be undefined, in which case the parametric form is better

(3) each point on L is uniquely associated with a value of the parameter t

(4) the equations of a particular line are not unique e.g. in above example

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