Sequences and Series

Geometric Sequences

Consider the sequence 1, 2, 4, 8, 16, ...

The first term is 1 and the terms are multiplied by 2 each time.

This is a **geometric sequence** with first term $a = 1$ and common ratio $r = 2$.

The n^{th} term is denoted u_n and in general:

$$u_n = ar^{n-1}$$

Example 1

Find the 10^{th} term of the geometric sequence 7, 14, 28, 56, . . .

$a = 7, r = 2$

$$u_n = ar^{n-1}$$
$$u_{10} = ar^9$$
$$u_{10} = 7 \times 2^9$$
$$= \underline{\underline{3584}}$$

Example 2

Find the 7^{th} term of the geometric sequence 45, -30, 20, -13 ⅓

$$a = 45, r = \frac{-30}{45} = \frac{-2}{3}$$

$$u_n = ar^{n-1}$$

$$u_7 = ar^6$$
$$= 45 \times \left(\frac{-2}{3}\right)^6$$
$$= 3\frac{77}{81}$$

Example 3

A geometric sequence of positive terms has 3rd term 72 and 7th term 4½. Find the 6th term of this sequence.

$$u_n = ar^{n-1}$$

$u_3 = 72 \quad \Rightarrow ar^2 = 72 \quad (1)$

$u_7 = 4\frac{1}{2} \quad \Rightarrow ar^6 = 4\frac{1}{2} \quad (2)$

(2) ÷ (1) $\quad \dfrac{ar^6}{ar^2} = \dfrac{4\frac{1}{2}}{72}$

$r^4 = \dfrac{1}{16} \quad \Rightarrow r = \pm \frac{1}{2}$

But, $r \neq -\frac{1}{2}$ since all terms are positive hence $r = \frac{1}{2}$.

Sub $r = \frac{1}{2}$ in (1) $\quad ar^2 = 72$

$a \times (\frac{1}{2})^2 = 72$

$a = \dfrac{72}{\frac{1}{4}} = 288$

$u_6 = ar^5$

$= 288 \times (\frac{1}{2})^5$

$= 9$