Algebraic Skills

Finding Powers of Real Numbers using the Binomial Theorem

The technique is to split the real number, R into two parts x and y

where x is the closest integer to R and y is the remaining part.

R = (x + y)^n or (x - y)^n

R = 1.9
R = (2 - 0.1)

R = 1.04
R = (1 + 0.04)

Examples

Use the Binomial Theorem to find

(a) $(1.02)^3$ (b) $(0.9)^7$ (c) $(2.1)^4$

$(1.02)^3 = (1 + 0.02)^3 \quad x = 1, y = 0.02$

$= 1 \cdot 1^3 + 3 \cdot 1^2 \cdot (0.02)^1 + 3 \cdot 1 \cdot (0.02)^2 + 1 \cdot (0.02)^3$

$= 1 + 0.06 + 0.00012 + 0.000008$

$= \underline{\underline{1.061}}$ to 3 d.p.

Note that the 0.02^3 does not contribute to the rounded result.

$(0.9)^7 = (1 - 0.1)^7 \quad x = 1, y = -0.1$

$= 1 \cdot 1^7 + 7 \cdot 1^6 \cdot (-0.1)^1 + 21 \cdot 1^5 \cdot (-0.1)^2$
$+ 35 \cdot 1^4 \cdot (-0.1)^3$..

$= 1 - 0.7 + 0.21 - 0.035 \ldots$

$= \underline{\underline{0.48}}$ to 2 d.p.

Note that the terms in $(-0.1)^4$ and higher do not contribute to the rounded result.

$(2.1)^4 = (2 + 0.1)^4 \quad x = 2, y = 0.1$

$= 1 \cdot 2^4 + 4 \cdot 2^3 \cdot (0.1)^1 + 6 \cdot 2^2 \cdot (0.1)^2 + 4 \cdot 2^1 \cdot (0.1)^3 \ldots$

$= 16 + 3.2 + 0.24 + 0.008$

$= \underline{19.45}$ to 2 d.p.

Note that the 0.1^4 does not contribute to the rounded result.

2009 Q8

(*a*) Write down the binomial expansion of $(1 + x)^5$.

(*b*) Hence show that $0{\cdot}9^5$ is $0{\cdot}59049$.

(a) $(1 + x)^5$

$= \underline{\underline{1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5}}$

(b) let $x = -0.1$

$0.9^5 = (1 + (-0.1))^5$

$= 1 - 0.5 + 10(-0.1)^2 + 10(-0.1)^3 + 5(-0.1)^4 + (-0.1)^5$

$= 1 - 0.5 + 0.1 - 0.01 + 0.0005 - 0.00001$

$= \underline{\underline{0.59049}}$ as req^d