Complex Numbers

De Moivres Theorem

$z = r[\cos\theta + i\sin\theta]$ *be any general complex number*

$z^2 = r^2\left[\cos 2\theta + i\sin 2\theta\right]$

$z^3 = r^3\left[\cos 3\theta + i\sin 3\theta\right]$

In general

De Moivres theorem $z^n = r^n[\cos n\theta + i\sin n\theta]$

Valid for positive, negative and fractional powers of complex numbers in polar form.

Examples

$z = 8[\cos 50 + i\sin 50]$ $w = 2[\cos 30 + i\sin 30]$

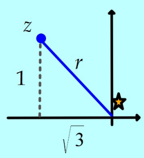
$w^3 = 2^3[\cos(30\times 3) + i\sin(30\times 3)]$
$w^3 = \underline{8[\cos 90 + i\sin 90]}$

$z^2 = 2^2[\cos(50\times 2) + i\sin(50\times 2)]$
$z^2 = \underline{4[\cos 100 + i\sin 100]}$

$\frac{w^3}{z^2} = \underline{\tfrac{1}{8}\left[\cos(-10) + i\sin(-10)\right]}$

Examples

Write $z = -\sqrt{3} + i$ in polar form.
Hence (a) write/express z^4 in the form $x + yi$
(b) show $z^6 + 64 = 0$



$r^2 = 1^2 + \sqrt{3}^2$ $\tan(\theta) = -\frac{1}{\sqrt{3}}$

$r = \sqrt{4}$ $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ *2nd Quadrant*

$r = 2$ $\theta = 150°$

$z = r(\cos\theta + i\sin\theta)$
$\underline{z = 2(\cos 150 + i\sin 150)}$

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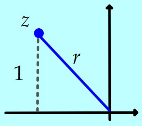

$$z = 2(\cos 150 + i\sin 150)$$

$$z^4 = 2^4\left(\cos(4 \times 150) + i\sin(4 \times 150)\right)$$

$-180 < \theta < 180$

$$z^4 = 16(\cos(600) + i\sin(600))$$

$$z^4 = 16(\cos(-120) + i\sin(-120))$$

$$z^4 = 16\left(-\tfrac{1}{2} - \tfrac{\sqrt{3}}{2}i\right)$$

$$\underline{\underline{z^4 = -8 - 8\sqrt{3}i}}$$

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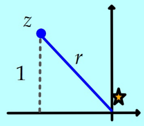

$$z = 2(\cos 150 + i\sin 150)$$

$$z^6 = 2^6\left(\cos(6 \times 150) + i\sin(6 \times 150)\right)$$

$$z^6 = 64(\cos(900) + i\sin(900))$$

$$z^6 = 64(\cos(-180) + i\sin(-180))$$

$$z^6 = 16(-1 - 0i)$$

$$\underline{\underline{z^6 = -64}}$$

Hence $z^6 + 64$
$= -64 + 64$
$= 0$ *as required*