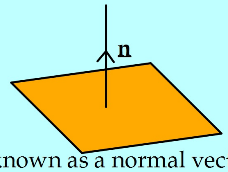
Vectors

The Equation of a Plane

A plane is simply a flat, 2D surface.



A vector which is ⊥ to a plane is known as a normal vector and is denoted $\underline{n}$.

A normal vector is in fact ⊥ to all vectors in a plane.

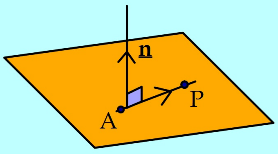
The Equation of a Plane

Consider the plane π.

Let A be a fixed point in this plane.

And let $\underline{n}$ be a vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ that is normal to the plane and passes through A.

Finally, let P (x, y, z) be a typical point in the plane (see below).



$\overrightarrow{AP}$ ⊥ to $\underline{n}$ $\Rightarrow$ $\underline{n}.\overrightarrow{AP} = 0$

$$\underline{n}.(\underline{p} - \underline{a}) = 0$$

$$\underline{n}.\underline{p} - \underline{n}.\underline{a} = 0$$

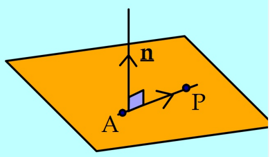
$$\underline{n}.\underline{p} = \underline{n}.\underline{a}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix}.\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}.\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$$

a random, unknown point

$ax + by + cz =$ [some constant]

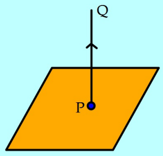
$ax + by + cz = k$ (since components of $\underline{n}$ and $\underline{a}$ are known)



∴ to find the equation of a plane you need a point on the plane and its normal vector.

Example

Find the equation of the plane perp. to PQ which contains P, where P is (1, 2, 3) and Q is (2, 1, -4).



Normal = $\overrightarrow{PQ} = \underline{q} - \underline{p} = \begin{pmatrix} 1 \\ -1 \\ 7 \end{pmatrix}$

Point = P(1, 2, 3) ***why not use Q?***

Equation is $\underline{n}.\underline{p} = \underline{n}.\underline{a}$

$$\begin{pmatrix} 1 \\ -1 \\ -7 \end{pmatrix}.\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -7 \end{pmatrix}.\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$x - y - 7z = 1 - 2 - 21$

$x - y - 7z = -22$

EX6 PAGE 57
Q1ab, 2ab

Example

Find the equation of the plane ⊥ to the vector $\underline{i} - 3\underline{j} + 2\underline{k}$ and containing the point P(-1, 2, 1).

Normal = $\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$

Point = P(-1, 2, 1)

Equation is $\underline{n}.\underline{p} = \underline{n}.\underline{a}$

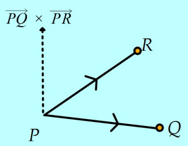
$$\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}.\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}.\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

$x - 3y + 2z = -1 - 6 + 2$

$x - 3y + 2z = -5$

Example

Find the equation of the plane which passes through the points A(-2, 1, 2), B(0, 2, 5) and C(2, -1, 3).



Normal = $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & 3 \\ 4 & -2 & 1 \end{vmatrix}$

$= 7\underline{i} + 10\underline{j} - 8\underline{k}$

Point = A(-2, 1, 2)

Equation is $\underline{n}.\underline{p} = \underline{n}.\underline{a}$

$$\begin{pmatrix} 7 \\ 10 \\ -8 \end{pmatrix}.\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 10 \\ -8 \end{pmatrix}.\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$$

$7x + 10y - 8z = -14 + 10 - 16$

$7x + 10y - 8z = -20$

EX6 PAGE 57/58
Q3, 4AC, 5A, 9, 10

Example

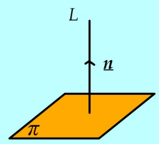
The equation of a line L is given by:

$$\frac{x}{-2} = \frac{y+2}{-1} = \frac{z-9}{2}$$

numerator = point on line
denominator = direction vector

The plane π is ⊥ to the line L and passes through the point (1, -4, 2).

What is the equation of the plane?



Normal: $\begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$

Point: (1, -4, 2)

Equation: $2x + y - 2z = -6$

Example

Obtain an equation for the plane containing the lines:

$$L_1: \; x = 1 + 2\lambda, \quad y = 2 - \lambda, \quad z = 1 - 2\lambda$$
$$L_2: \; x = 1 - \mu, \quad y = 2 + \mu, \quad z = 1 + 3\mu$$

Intersection point of these two lines: (1, 2, 1)
HINT: equate x's & y's then solve simultaneously

Normal vector to the plane: $-\underline{i} - 4\underline{j} + \underline{k}$
HINT: vector product

Equation of Plane: $x + 4y - z = 8$

2013 Q15 6 marks
2012 Q5 5 marks
2008 Q14 3 marks