Functions

Odd and Even Functions

Even

Produces same answers on both sides.

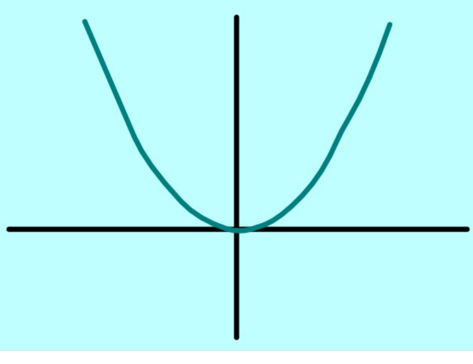
1. $f(-x) = f(x)$
2. curve symmetrical about y-axis

Example

$f(x) = x^2$ is an even function.

1. $f(-x) = (-x)^2$
$= x^2$
$= f(x)$

2. $f(x) = x^2$



Odd and Even Functions

Odd

Produces same answers on both sides only one positive and one negative.

1. $f(-x) = -f(x)$
2. curve has half turn symmetry about the Origin

Example

$f(x) = x^3$ is an odd function.

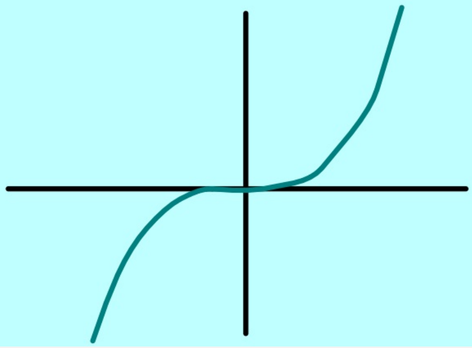
1. $f(-x) = (-x)^3$

$= -x^3$

$= \underline{\underline{-f(x)}}$

2.

$f(x) = x^3$



For polynomial functions:

All ODD powers → ODD function: e.g. $x^3 + 2x$

All EVEN powers → EVEN function: e.g. $x^6 - 2x^2 + 3$

e.g. $f(x) = \cos x + x^2$: f is even.
$g(x) = x + \sin x$: g is odd.

$h(x) = x^3 + x^2 + x$: h is neither even nor odd, but it is the sum of an even function (x^2) and an odd function ($x^3 + x$).

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\tan(-x) = -\tan x$$

Example

Investigate whether the function $f(x) = x^3 \sin x$ is Odd or Even.

$$f(-x) = (-x)^3 \sin(-x)$$
$$f(-x) = -x^3 \times -\sin x$$
$$f(-x) = x^3 \sin x$$

Hence $f(-x) = f(x)$ so f is an Even function.