Vectors

Angle between Two Lines

θ = angle between the direction vectors of the lines.

(1) Lines must intersect

(2) Express lines in symmetric form

(3) Use the scalar product to find θ.

Note/ The angle between the lines is the angle between their direction vectors. However, this angle is usually taken as acute, so if obtuse angle results, use its supplement.

Example

Find the size of the angle between the lines:

$x - 1 = y = z - 1$ and $x = 1 + t$, $y = 5t$, $z = -t$

The lines can be expressed in symmetric form i.e.

$$\frac{x-1}{1} = \frac{y}{1} = \frac{z-1}{1} \quad \text{and} \quad \frac{x-1}{1} = \frac{y}{5} = \frac{z}{-1}$$

Their direction vectors are $\underline{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix}$

$$\cos(\theta) = \frac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|} = \frac{1+5-1}{\sqrt{3}\sqrt{27}} = \frac{5}{9} \quad \Rightarrow \theta = 56.3°$$

ex 11 page 70 Q1(iii), 2(b)(ii)

2007 Q15 3 marks
2011 Q15b 4 marks