Sequences and Series

Infinite Geometric Series

$$S_\infty = \frac{a}{1-r}, \quad -1 < r < 1$$

Example 1

Find the sum of the first 9 terms of the geometric series

$$4 + 8 + 16 + 32 + \ldots$$

$a = 4$, r = 2 and n = 9

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_9 = \frac{4(2^9 - 1)}{2 - 1}$$

$$S_9 = 4(2^9 - 1)$$

$$S_9 = 2044$$

Example 1

Find the sum to infinity of the geometric series

$$250 + 150 + 90 + 54 + \ldots$$

$a = 250 \qquad r = \frac{150}{250} = \frac{3}{5}$

$-1 < r < 1$, so a sum to infinity exists.

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{250}{1 - \frac{3}{5}}$$

$$= 625$$

Example 2

A geometric series has first term 28 and sum to infinity 16.
Find the 4th term of the sequence.

$u_1 = 28 \Rightarrow a = 28$

$S_\infty = 16 \Rightarrow \frac{a}{1-r} = 16$

$$\frac{28}{1-r} = 16$$

$$\frac{28}{16} = 1 - r$$

$$r = -\frac{3}{4}$$

(note a sum to infinity exists as $-1 < r < 1$)

$$u_4 = ar^3$$

$$= 28 \times \left(\frac{-3}{4}\right)^3$$

$$= -11\frac{13}{16}$$

DON'T FORGET

$$\sum_{n=k}^{m} = \sum_{n=1}^{m} - \sum_{n=1}^{k-1}$$ applies to all series.

You will use it when you want to find the sum of terms which start part-way along the series.

$$\sum_{k=1}^{n} c = nc$$ for any constant c.

Example 3

Evaluate the sum $\sum_{r=10}^{20}(3r-5)$

$$\sum_{r=10}^{20}(3r-5) = \left(3\sum_{r=1}^{20} r - \sum_{r=1}^{20} 5\right) - \left(3\sum_{r=1}^{9} r - \sum_{r=1}^{9} 5\right)$$

$$= \left(\frac{3}{2} \times 20(21) - 5 \times 20\right) - \left(\frac{3}{2} \times 9(10) - 6 \times 9\right)$$

$$= 530 - 90$$

$$= 440$$

DON'T FORGET

The square and cube numbers do not form arithmetic or geometric sequences.

However, the formula for the sum of their terms is known.

Remember these results:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$