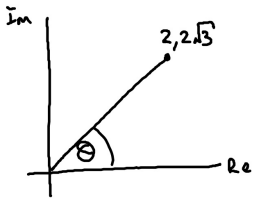
Finding n^{th} roots of complex numbers

- Write complex numbers in Polar Form.
- $z^n = r^n(\cos n\theta + i\sin n\theta)$
  Find z using De Moivres Thm
- Use an Argand Diagram to find other roots.

Example 2

$2 + 2\sqrt{3}\,i$



$r = \sqrt{2^2 + (2\sqrt{3})^2}$

$= 4$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right)$

$= 60°$

since "Square roots"

$z^2 = 4(\cos 60 + i\sin 60)$

First Root

$z^2 = 4(\cos 60 + i\sin 60)$

$z = 4^{\frac{1}{2}}(\cos 60 + i\sin 60)^{1/2}$

$= 2(\cos 30 + i\sin 30)$

Second Root

$z^2 = 4(\cos 60 + i\sin 60)$

$= 4(\cos(60+360) + i\sin(60+360))$

$= 4(\cos 420 + i\sin 420)$

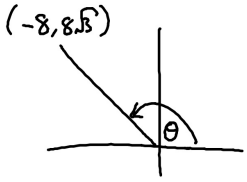
$z = 4^{\frac{1}{2}}(\cos 420 + i\sin 420)^{\frac{1}{2}}$

$= 2(\cos 210 + i\sin 210)$

$= 2(\cos(-150) + i\sin(-150))$

Example 3

$-8+8\sqrt{3}i$



$r = \sqrt{8^2 + (8\sqrt{3})^2}$

$= 16$

$\theta = \tan^{-1}\left(\frac{8\sqrt{3}}{8}\right)$

$= 60^\circ$

$\Rightarrow 120^\circ$

$z^2 = 16(\cos 120 + i\sin 120)$

First Root

$z = 16^{\frac{1}{2}}(\cos 120 + i\sin 120)^{\frac{1}{2}}$

$= 4(\cos 60 + i\sin 60)$

Second Root

$z = 4(\cos 60 + i\sin 60)$

The second root is 'equally spaced'

$60^\circ + 180^\circ = 240^\circ$ ✗ outwith $-180 < \theta < 180$

$60 - 180^\circ = -120^\circ$ ✓ * since $\frac{360}{2} = 180^\circ$

$z = 4(\cos(-120) + i\sin(-120))$

NB The roots of a complex number are found equally spaced around the Argand Diagram.

We can use this fact to find other roots once we have found the 1st root.

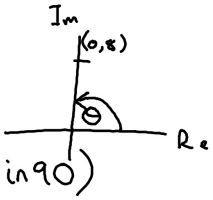
e.g. 'square roots' are 180° apart $(360 \div 2)$

'cube roots' are 120° apart $(360 \div 3)$

Remember $-180 < \theta < 180$

Example 4

$8i$ or $0+8i$



$z^3 = 8(\cos 90 + i\sin 90)$

First Root

$z = 8^{\frac{1}{3}}(\cos 90 + i\sin 90)^{\frac{1}{3}}$

$= 2(\cos 30 + i\sin 30)$

3 roots are equally spaced: $\frac{360}{3} = 120$

Second Root $2(\cos 150 + i\sin 150)$

Third Root $2(\cos(-90) + i\sin(-90))$

EXTENSION

Write these roots in Cartesian Form

$(a+bi)(x+yi)$