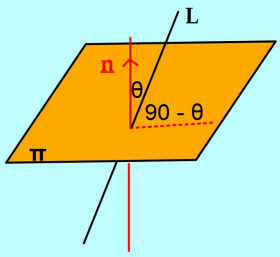
Vectors

Angle between a Line and a Plane

Assume that the line is not parallel to the plane.

If θ is the angle between the line and the normal vector to the plane, then (90 - θ) is the angle between the line and the line.



Example

Find the size of the angle between the line

$\frac{x-1}{2} = \frac{y}{1} = \frac{z+2}{-1}$ and the plane $2x + 3y - z = 0$

Direction Vector of Line = $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \underline{l}$

Normal Vector of Plane = $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \underline{n}$

$\cos(\theta) = \frac{\underline{l}.\underline{n}}{|\underline{l}||\underline{n}|} = \frac{4+3+1}{\sqrt{6}\sqrt{14}} = \frac{8}{\sqrt{84}} \Rightarrow \theta = 29.2°$

So angle between line and plane = 90 - 29.2 = 60.8°

Example

Find the size of the angle between the line

$x = 2 - t, \quad y = -2t, \quad z = 1 + 2t$ and the plane $x + y + z = 0$

Direction Vector of Line = $\begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} = \underline{l}$

Normal Vector of Plane = $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \underline{n}$

$\cos(\theta) = \frac{\underline{l}.\underline{n}}{|\underline{l}||\underline{n}|} = \frac{-1-2+2}{\sqrt{3}\sqrt{9}} = \frac{-1}{3\sqrt{3}} \Rightarrow \theta = 101.1°$

The acute angle between the line and normal vector to the plane is 180 - 101.1 = 78.9

∴the angle between the line and plane is 90 - 78.9 = 11.1

ex10 page 68 Q1(ii)abc, 2ab, 3, 4a

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