Complex Numbers

Geometrical Interpretation of Equations and Inequalities in the Complex Plane

The solution set of complex equations and inequalities can be represented by sets of points in an Argand Diagram.

Examples

Given that $z = x + yi$, draw the locus of the point which moves on the complex plane so that:

1 $|z| = 1$

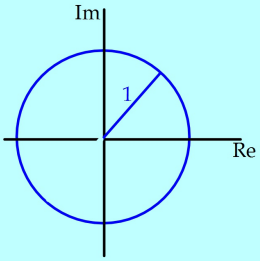
2 (a) $|z - 1| = 2$ **2 (b)** $|z - 1| \leq 2$

3 $|z - 2| = |z + 2i|$

1 $|z| = 1 \Rightarrow \sqrt{x^2 + y^2} = 1$

i.e. $x^2 + y^2 = 1$

This is a circle, centre the Origin and radius 1

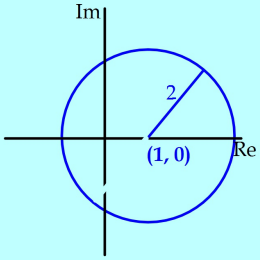


2 (a) $|z - 1| = 2 \Rightarrow |(x - 1) + yi| = 2$

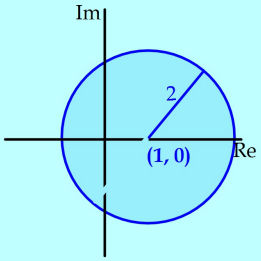
$\sqrt{(x - 1)^2 + y^2} = 2$

$(x - 1)^2 + y^2 = 4$

This is a circle, centre (1, 0) and radius 2



2 (b) $|z-1| \leq 2$ is the set of points inside or on the circumference of the circle with centre (1, 0) and radius 2.



3 $|z-2| = |z+2i|$

$\Leftrightarrow |(x-2)+yi| = |x+i(y+2)|$

$\Rightarrow \sqrt{(x-2)^2+y^2} = \sqrt{x^2+(y+2)^2}$

$(x-2)^2+y^2 = x^2+(y+2)^2$

$x^2-4x+4+y^2 = x^2+y^2+4y+4$

$-4x = 4y$

$y = -x$

This is a straight line through the origin with gradient -1.