

# Vectors

## Intersection of Two Lines

Two lines in space can

- (i) intersect at a point
- or (ii) be parallel
- or (iii) be skew (not parallel but will never intersect)

### To find point of intersection

- (1) Express equations in parametric form (parameters  $t_1$  &  $t_2$ )
- (2) Equate corresponding expressions for  $x$ ,  $y$ , and  $z$  (3 equations in  $t_1$  &  $t_2$ )
- (3) Use two of the equations to find  $t_1$  &  $t_2$
- (4) Substitute values of  $t_1$  &  $t_2$  into 3rd equation.  
If they satisfy the equation then the point of intersection has been found; if they don't then the lines do not intersect.

### Example

Find the point of intersection of the lines

$$x - 5 = -(y + 2) = z \quad \text{and} \quad \frac{x-12}{5} = \frac{y+3}{-2} = \frac{z-5}{4}$$

$$L_1: \quad x = t_1 + 5, \quad y = -t_1 - 2, \quad z = t_1$$

$$L_2: \quad x = 5t_2 + 12, \quad y = -2t_2 - 3, \quad z = 4t_2 + 5$$

Equating corresponding coordinates:

$$t_1 + 5 = 5t_2 + 12 \quad \textcircled{1}$$

$$-t_1 - 2 = -2t_2 - 3 \quad \textcircled{2}$$

$$t_1 = 4t_2 + 5 \quad \textcircled{3}$$

$$t_1 + 5 = 5t_2 + 12 \quad \textcircled{1}$$

$$-t_1 - 2 = -2t_2 - 3 \quad \textcircled{2}$$

$$t_1 = 4t_2 + 5 \quad \textcircled{3}$$

$$\begin{aligned} \textcircled{1} + \textcircled{2} &=> 3 = 3t_2 + 9 \\ -6 &= 3t_2 \\ t_2 &= -2 \end{aligned}$$

$$\begin{aligned} \text{Subst } t_2 = -2 \text{ in } \textcircled{1} \quad t_1 + 5 &= -10 + 12 \\ t_1 &= -3 \end{aligned}$$

**Check/** Subst into  $\textcircled{3}$   $-3 = 4(-2) + 5$   
Hence lines meet where  $t_1 = -3$  &  $t_2 = -2$

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Hence lines meet where  $t_1 = -3$  &  $t_2 = -2$

$$L_1: x = t_1 + 5, y = -t_1 - 2, z = t_1$$

$$\begin{aligned} x &= -3 + 5, y = -(-3) - 2, z = -3 \\ &= 2 \qquad \qquad = 1 \end{aligned}$$

$$L_2: x = 5t_2 + 12, y = -2t_2 - 3, z = 4t_2 + 5$$

$$\begin{aligned} x &= 5(-2) + 12, y = -2(-2) - 3, z = 4(-2) + 5 \\ &= 2 \qquad \qquad = 1 \qquad \qquad = -3 \end{aligned}$$

The point of intersection is (2, 1, -3)

ex11 page 70 Q1(i)(ii), 2ab(i)

**2007 Q15 3 marks**  
**2011 Q15 6 marks**

