Curve Sketching

Sketching curves

Method

(a) Find all the asymptotes and investigate the approach of the curve to each.

(b) Find all SPs and determine their nature.

(c) Find all crossings of the y-axis and the x-axis (if easily found)

(d) Sketch the curve

Note: The nature table may be easier than finding the second derivative in order to determine the SPs.

Example

Sketch the curve of $f(x) = \dfrac{2x^2 + x - 1}{x - 1}$

$$f(x) = \frac{(x+1)(2x-1)}{x-1}$$

(a) Asymptotes

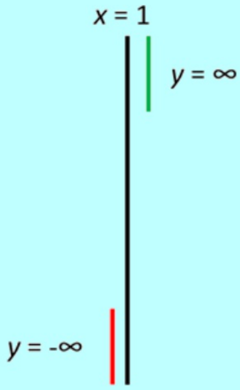
(i) Vertical Asymptotes ***occur when the denominator equal zero***

$x - 1 = 0$

$x = 1$

As $x \to 1^-$ $\quad f(x) = \dfrac{(+)(+)}{(-)} \quad \to -\infty$

As $x \to 1^+$ $\quad f(x) = \dfrac{(+)(+)}{(+)} \quad \to \infty$



(ii) Non Vertical Asymptotes

Divide the numerator by the denominator using long division

$$\begin{array}{r|rrrr} & & 2x & + & 3 \\ \hline x - 1 & 2x^2 & + x & - & 1 \\ - & 2x^2 & - 2x & & \downarrow \\ \hline & & 3x & - & 1 \\ - & & 3x & - & 3 \\ \hline & & & & 2 \end{array}$$

$$Q + \frac{r}{d}$$

$$f(x) = 2x + 3 + \frac{2}{x-1}$$

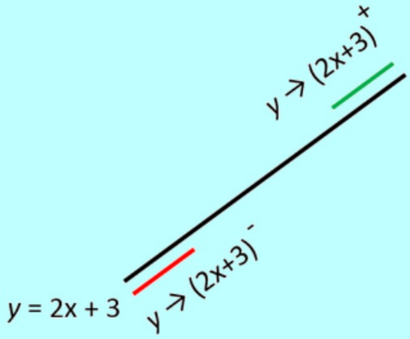
Divide the fraction by the highest power of *x*. $\quad f(x) = 2x + 3 + \dfrac{\frac{2}{x}}{\frac{x}{x} - \frac{1}{x}}$

$$f(x) = 2x + 3 + \frac{\frac{2}{x}}{1 - \frac{1}{x}}$$

As $x \to \pm\infty$, $f(x) \to 2x + 3$ and so $f(x) = 2x + 3$ is a slant or oblique asymptote.

As $x \to +\infty$ $y \to (2x + 3)^+$

As $x \to -\infty$ $y \to (2x + 3)^-$



(b) Stationary Points

$$f(x) = 2x + 3 + \frac{2}{x-1}$$

$$f'(x) = 2 - \frac{2}{(x-1)^2}$$

$$f'(x) = 0 \quad @\ \text{SP}$$

$$2 - \frac{2}{(x-1)^2} = 0$$

$$\frac{2}{(x-1)^2} = 2$$

$$(x-1)^2 = 1$$

$$x = 0 \text{ or } 2$$

When $x = 0$, $f(x) = 1$ i.e. (0, 1)

When x =2, $f(x) = 9$ i.e. (2, 9)

Nature

$$f''(x) = \frac{4}{(x-1)^3}$$

$$f''(0) = \frac{4}{(-)} = (-) \quad \therefore \quad (0,\ 1) \text{ is a max TP}$$

$$f''(2) = \frac{4}{(+)} = (+) \quad \therefore \quad (2,\ 9) \text{ is a min TP}$$

(c) Axes Crossing

Cuts x-axis when $f(x) = 0$

$$0 = \frac{(x+1)(2x-1)}{x-1}$$

$$0 = (x+1)(2x-1)$$

$x = -1$ and x = ½ i.e. (-1, 0) and (½ , 0)

Cuts y-axis when $x = 0$

$$f(0) = \frac{(0+1)(2(0)-1)}{0-1} = 1 \qquad \text{i.e. } (0,\ 1)$$

(d) Sketch