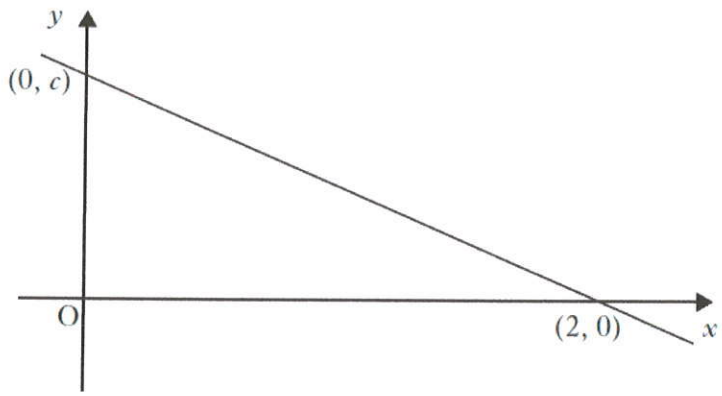
Curve Sketching

2013

Q13 – 5 marks

Part of the straight line graph of a function $f(x)$ is shown.



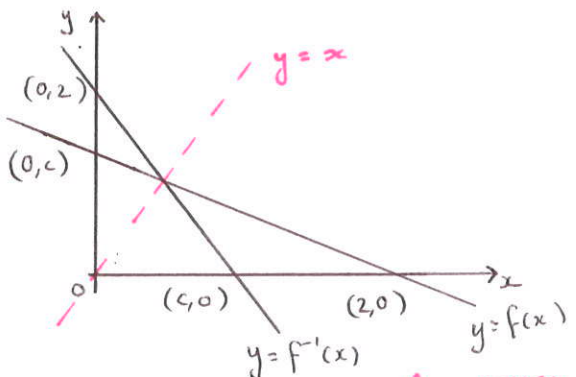
(a) Sketch the graph of $f^{-1}(x)$, showing points of intersection with the axes. **2**

(b) State the value of k for which $f(x) + k$ is an odd function. **1**

(c) Find the value of h for which $|f(x + h)|$ is an even function. **2**

Written Solutions

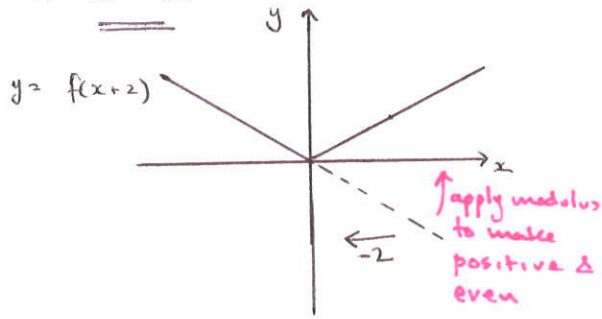
(a)



To find the graph of inverses, you reflect in the line $y = x$

(b) $f(x) + k$ is odd $\Rightarrow$ $k = -c$

(c) $|f(x+h)|$ is even when $h = 2$



An "even fn" is any fn whose curve has the y-axis as their line of symmetry

An "odd fn" is a fn whose curve has a 180° symmetry about the origin

2012

Q7 – marks

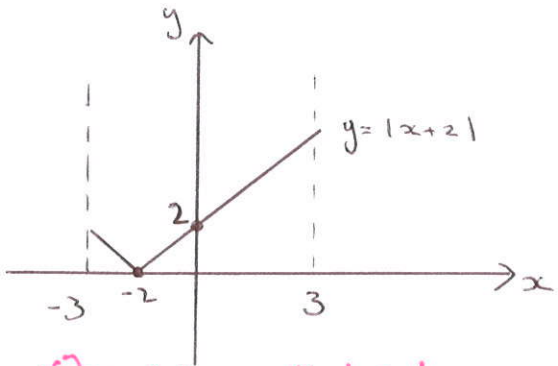
A function is defined by $f(x) = |x + 2|$ for all x.

(*a*) Sketch the graph of the function for $-3 \leq x \leq 3$. 2

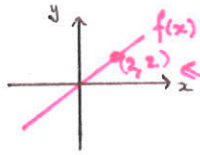
(*b*) On a separate diagram, sketch the graph of $f'(x)$. 2

Written Solutions

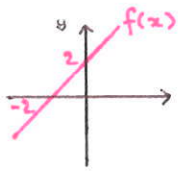
(a)



modulus applied and makes positive.

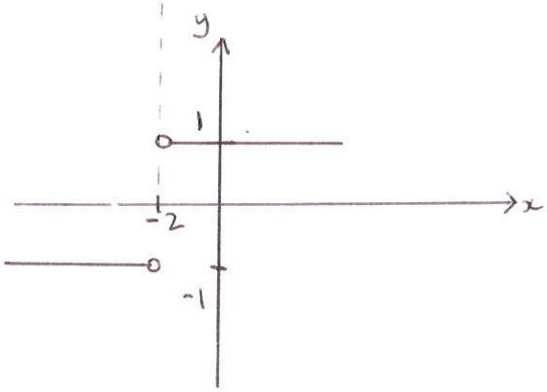


just a 'random' example for this explanation



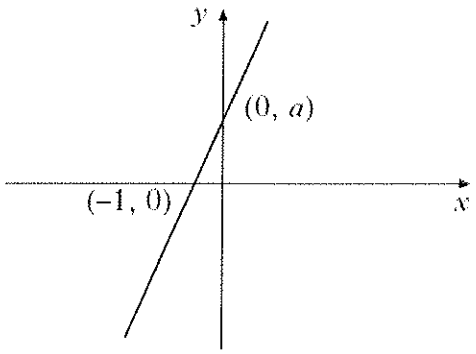
in brackets since inside modulus.

(b)



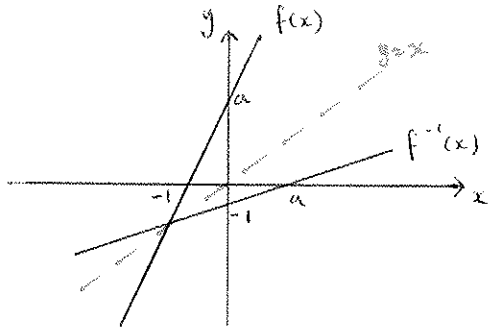
2011

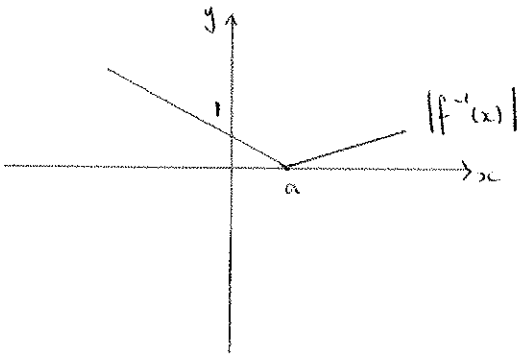
Q6 – 4 marks



The diagram shows part of the graph of a function $f(x)$. Sketch the graph of $|f^{-1}(x)|$ showing the points of intersection with the axes. **4**

Written Solutions

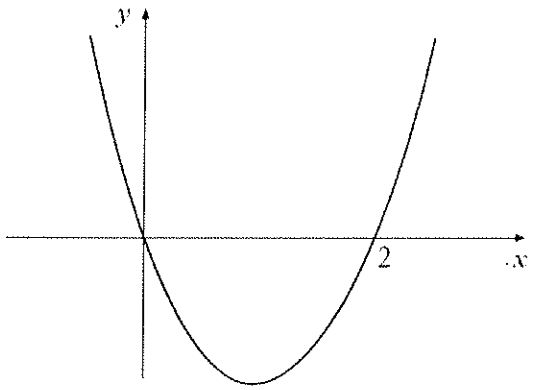




2010

Q10 – 3 marks

The diagram below shows part of the graph of a function $f(x)$. State whether $f(x)$ is odd, even or neither. Fully justify your answer.



3

Written Solutions

The graph is not symmetrical about the y-axis $\therefore$ not even

The graph is not odd since no half turn rotational symmetry.

$\therefore$ The f/n is neither even nor odd.

2009

Q13 – 10 marks

The function $f(x)$ is defined by

$$f(x) = \frac{x^2 + 2x}{x^2 - 1} \qquad (x \neq \pm 1).$$

Obtain equations for the asymptotes of the graph of $f(x)$. **3**

Show that $f(x)$ is a strictly decreasing function. **3**

Find the coordinates of the points where the graph of $f(x)$ crosses

(i) the x-axis and

(ii) the horizontal asymptote. **2**

Sketch the graph of $f(x)$, showing clearly all relevant features. **2**

Written Solutions

$f(x) = \frac{x^2+2x}{x^2-1} = \frac{x(x+2)}{(x+1)(x-1)}$

Asymptotes

Vertical: occur when denominator = 0

i.e. $x = -1$ and $x = 1$

As $x \to -1^-$ $\quad y \to \frac{(-)(+)}{(-)(-)} \to -\infty$

As $x \to -1^+$ $\quad y \to \frac{(-)(+)}{(+)(-1)} \to +\infty$

As $x \to 1^-$ $\quad y \to \frac{(+)(+)}{(+)(-)} \to -\infty$

As $x \to 1^+$ $\quad y \to \frac{(+)(+)}{(+)(+)} \to +\infty$

Non-vertical:

$$x^2 - 1 \;\overline{)\; x^2 + 2x + 0 \;\big|\; 1}$$

$$\underline{-\,(x^2 \qquad -1)}$$

$$2x + 1$$

So $f(x) = 1 + \frac{2x+1}{x^2-1}$

$$= 1 + \frac{\frac{2x}{x^2} + \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}}$$

$$= 1 + \frac{\frac{2}{x} + \frac{1}{x^2}}{1 - \frac{1}{x^2}}$$

As $x \to \pm\infty$ $\quad f(x) \to 1$

and so $f(x) = 1$ is a horizontal asymptote.

As $x \to +\infty$ $\quad y \to 1^+$

As $x \to -\infty$ $\quad y \to 1^-$

$y = 1$

PTO

Stationery Points:

$$f(x) = 1 + \frac{2x+1}{x^2-1}$$

$$f'(x) = \frac{u'v - v'u}{v^2} \quad \text{where} \quad u = 2x+1 \quad u' = 2$$
$$v = x^2-1 \quad v' = 2x$$

$$= \frac{2(x^2-1) - 2x(2x+1)}{(x^2-1)^2}$$

$$= \frac{-2x^2-2x-2}{(x^2-1)^2}$$

$$= \frac{-2(x^2+x+1)}{(x^2-1)^2}$$

$\therefore$ $f'(x) < 0 \quad \forall x \Rightarrow$ strictly decreasing

(c) Axes crossing:

(i) cuts x-axis when $f(x) = 0$

$$0 = 1 + \frac{2x+1}{x^2-1}$$

$$\frac{2x+1}{x^2-1} = -1$$

$$2x+1 = -x^2+1$$

$$x(x+2) = 0$$

i.e. $x = 0$ or $x = -2$

(ii) cuts horizontal asymptote at $y = 1$

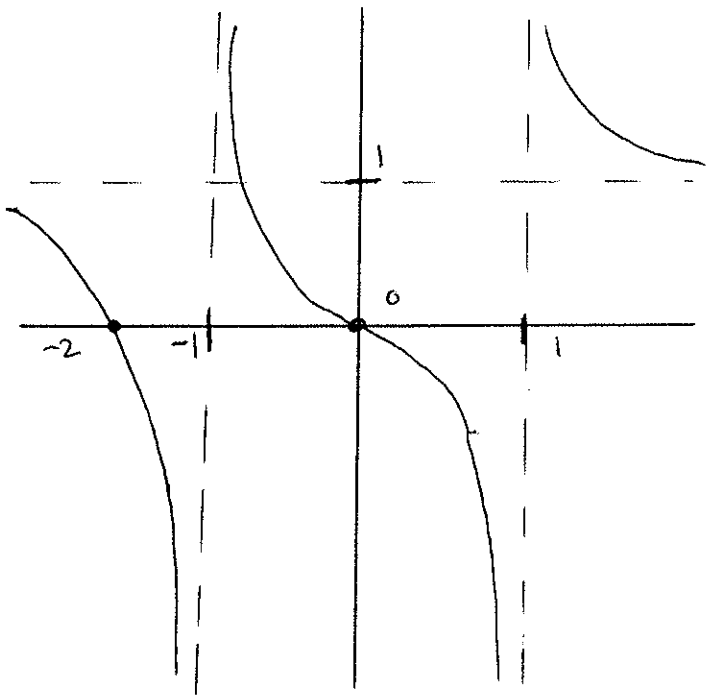
$$1 + \frac{2x+1}{x^2-1} = 1$$

$$\frac{2x+1}{x^2-1} = 0$$

$$2x+1 = 0$$

$$2x = -1$$

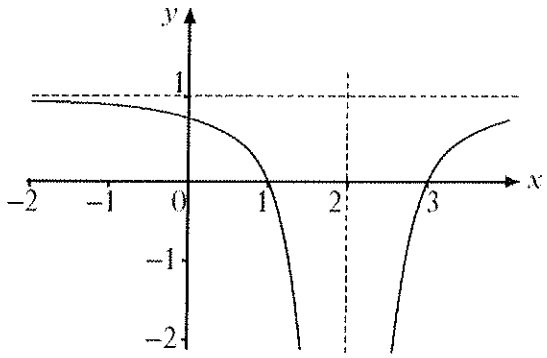
$$x = -\frac{1}{2}$$



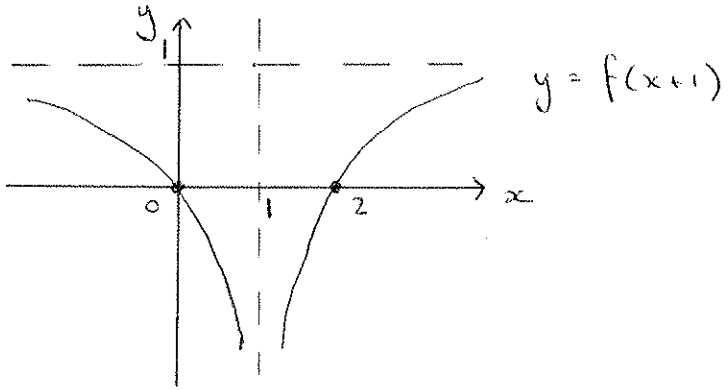
2008

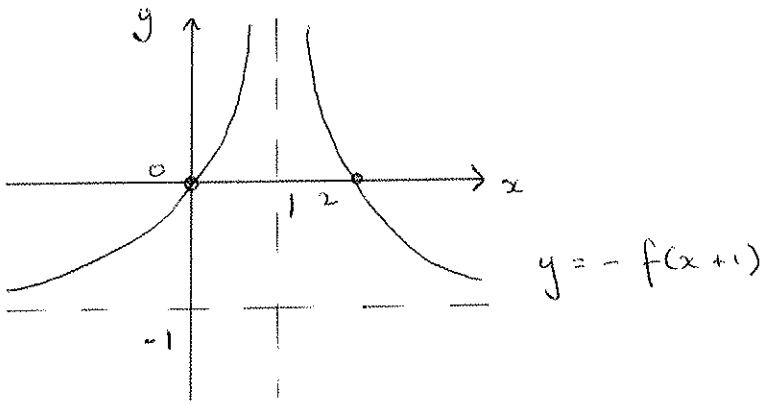
Q3 – 4 marks

Part of the graph $y = f(x)$ is shown below, where the dotted lines indicate asymptotes. Sketch the graph $y = -f(x + 1)$ showing its asymptotes. Write down the equations of the asymptotes. **4**



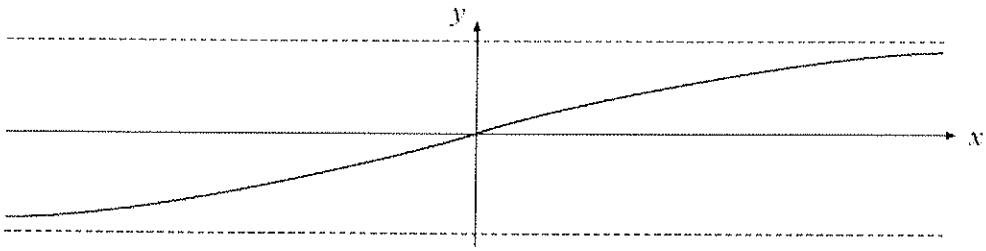
Written Solutions





2007

Q16 a & c – marks



(*a*) The diagram shows part of the graph of $f(x) = \tan^{-1} 2x$ and its asymptotes. State the equations of these asymptotes. 2

(*c*) Sketch the graph of $y = |f(x)|$ and calculate the area between this graph, the x-axis and the lines $x = -\frac{1}{2}$, $x = \frac{1}{2}$. 3

Written Solutions

(a)