Differentiation

2013

Q2 – 3 marks

Differentiate $f(x) = e^{\cos x}\sin^2 x$. **3**

Written Solutions

$$f(x) = e^{\cos x}\sin^2 x$$

$$f'(x) = e^{\cos x}\cdot(-\sin x)\cdot\sin^2 x + e^{\cos x}\cdot 2\sin x\cos x$$

$$= e^{\cos x}\cdot(-\sin^3 x) + e^{\cos x}\cdot \overset{*}{\sin} 2x$$

$$= \underline{\underline{e^{\cos x}\left(\sin 2x - \sin^3 x\right)}}$$

* $\sin 2x = 2\sin x\cos x$ from Higher

ALTERNATE (without *, also acceptable)

$$f(x) = e^{\cos x}\sin^2 x$$

$$f'(x) = e^{\cos x}\cdot(-\sin x)\cdot\sin^2 x + e^{\cos x}\cdot 2\sin x\cos x$$

$$= e^{\cos x}\cdot(-\sin^3 x) + e^{\cos x}\cdot 2\sin x\cos x$$

$$= \underline{\underline{e^{\cos x}\cdot\sin x\left(2\cos x - \sin^2 x\right)}}$$

2012
Q1 – 7 marks

(a) Given $f(x) = \dfrac{3x+1}{x^2+1}$, obtain $f'(x)$. **3**

(b) Let $g(x) = \cos^2 x \exp(\tan x)$. Obtain an expression for $g'(x)$ and simplify your answer. **4**

Written Solutions

(a) Quotient Rule

So $f'(x) = \dfrac{u'v - v'u}{v^2}$ where $u = 3x+1$, $u' = 3$, $v = x^2+1$, $v' = 2x$

$$= \frac{3(x^2+1) - 2x(3x+1)}{(x^2+1)^2}$$

$$= \underline{\underline{\frac{-3x^2 - 2x + 3}{(x^2+1)^2}}}$$

(b) Product Rule

So $g'(x) = u'v + v'u$ where $u = \cos^2 x$, $u' = -2\cos x \sin x$, $v = e^{\tan x}$, $v' = \sec^2 x\, e^{\tan x}$ *1

Hence $g'(x) = -2\cos x \sin x\, e^{\tan x} + \sec^2 x\, e^{\tan x} \cos^2 x$

$$= e^{\tan x}(-2\cos x \sin x + \sec^2 x \cos^2 x)$$

$$= \underline{\underline{e^{\tan x}(-\sin 2x + 1)}}$$ *2

*1 $f(x) = \tan x$ $f'(x) = \sec^2 x$

*2 $\sec^2 x = \dfrac{1}{\cos^2 x}$

2011
Q3b – 3 marks

(*b*) Given $f(x) = \sin x \cos^3 x$, obtain $f'(x)$. **3**

Written Solutions

$f(x) = \sin x \cos^3 x$ where $u = \sin x$ $u' = \cos x$

$v = \cos^3 x$ $v' = -3\cos^2 x \sin x$

$$f'(x) = u'v + v'u$$

$$= \cos x \cdot \cos^3 x + (-3\cos^2 x \sin x) \cdot \sin x$$

$$= \cos^4 x - 3\cos^2 x \sin^2 x$$

$$= \underline{\underline{\cos^2 x (\cos^2 x - 3\sin^2 x)}}$$

2011
Q7 – 4 marks

A curve is defined by the equation $y = \dfrac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$ for $x < 1$.

Calculate the gradient of the curve when $x = 0$. **4**

Written Solutions

$$y = \frac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}$$

$$\ln y = \ln\left(\frac{e^{\sin x}(2+x)^3}{\sqrt{1-x}}\right) \quad *1$$

$$\ln y = \ln\left(e^{\sin x}(2+x)^3\right) - \ln\left(\sqrt{1-x}\right)$$

$$\ln y = \ln\left(e^{\sin x}\right) + \ln(2+x)^3 - \ln\left((1-x)\right)^{1/2}$$

$$\ln y = \overset{*2}{\sin x} + 3\ln(2+x) - \frac{1}{2}\ln(1-x)$$

$$\frac{1}{y}\frac{dy}{dx} = \cos x + \frac{3}{2+x} + \frac{1}{2(1-x)}$$

$$\frac{dy}{dx} = y\left(\cos x + \frac{3}{2+x} + \frac{1}{2(1-x)}\right)$$

At $x = 0$, gradient of curve $= \dfrac{dy}{dx} = \dfrac{e^0 \cdot 2^3}{1}\left(1 + \dfrac{3}{2} + \dfrac{1}{2}\right)$

$$= \underline{\underline{24}}$$

*1 use rules of logs to simplify f^n before differentiation

*2 $\ln e = 1$

2010

Q1 – 6 marks

Differentiate the following functions.

(*a*) $f(x) = e^x \sin x^2$. **3**

(*b*) $g(x) = \dfrac{x^3}{(1+\tan x)}$. **3**

Written Solutions

(a) $f(x) = e^x \sin x^2$ where $u = e^x \quad u' = e^x$

$v = \sin x^2 \quad v' = \cos x^2 \cdot (2x)$

$$f'(x) = u'v + v'u$$

$$= e^x \cdot \sin x^2 + 2x\cos x^2 \cdot e^x$$

$$= \underline{\underline{e^x(\sin x^2 + 2x\cos x^2)}}$$

(b) $g(x) = \dfrac{x^3}{1+\tan x}$

So $g'(x) = \dfrac{u'v - v'u}{v^2}$ where $u = x^3 \quad u' = 3x^2$

$v = 1+\tan x \quad v' = \sec^2 x$

Hence $g'(x) = \underline{\underline{\dfrac{3x^2(1+\tan x) - \sec^2 x(x^3)}{(1+\tan x)^2}}}$

2009

Q1a – 3 marks

(*a*) Given $f(x) = (x + 1)(x - 2)^3$, obtain the values of x for which $f'(x) = 0$. **3**

Written Solutions

$f(x) = (x+1)(x-2)^3$

So $f'(x) = u'v + v'u$ where $u = x+1$, $u' = 1$

$v = (x-2)^3$, $v' = 3(x-2)^2$

Hence $f'(x) = (x-2)^3 + 3(x-2)^2(x+1)$

$= (x-2)^2\left[(x-2) + 3(x+1)\right]$

$= (x-2)^2(4x+1)$

$= 0$ when $\underline{\underline{x = 2}}$ and $\underline{\underline{x = -\frac{1}{4}}}$

2008

Q2a – 2 marks

(*a*) Differentiate $f(x) = \cos^{-1}(3x)$ where $-\frac{1}{3} < x < \frac{1}{3}$. **2**

Written Solutions

Note: Although an example of the Chain Rule we have not covered "differentiation of inverse functions" in Unit 1. Try using this:

$f(x)$	$f'(x)$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1-x^2}}$

$$f(x) = \cos^{-1}(3x)$$

$$f'(x) = -\frac{1}{\sqrt{1-(3x)^2}} \cdot 3$$

$$= -\frac{3}{\sqrt{1-9x^2}}$$

2008

Q15 – 9 marks

Let $f(x) = \dfrac{x}{\ln x}$ for $x > 1$.

(*a*) Derive expressions for $f'(x)$ and $f''(x)$, simplifying your answers. **2,2**

(*b*) Obtain the coordinates and nature of the stationary point of the curve $y = f(x)$. **3**

(*c*) Obtain the coordinates of the point of inflexion. **2**

Written Solutions

(a) $f'(x) = \dfrac{u'v - v'u}{v^2}$ where $u = x$, $u' = 1$, $v = \ln x$, $v' = \dfrac{1}{x}$

$$= \frac{\ln x - 1}{(\ln x)^2}$$

$f''(x) = \dfrac{u'v - v'u}{v^2}$ where $u = \ln x - 1$, $u' = \dfrac{1}{x}$, $v = (\ln x)^2$, $v' = \dfrac{2\ln x}{x}$

$$= \frac{\dfrac{(\ln x)^2}{x} - \dfrac{2\ln x(\ln x - 1)}{x}}{(\ln x)^4}$$

$$= \frac{(\ln x)^2 - 2\ln x(\ln x - 1)}{x(\ln x)^4}$$

$$= \frac{(\ln x)^2 - 2(\ln x)^2 + 2\ln x}{x(\ln x)^4}$$

$$= \frac{\ln x(\ln x - 2\ln x + 2)}{x(\ln x)^4}$$

$$= \frac{\ln x - 2\ln x + 2}{x(\ln x)^3}$$

$$= \frac{2 - \ln x}{x(\ln x)^3}$$

(b) $f'(x) = 0$ @ S.P.

So, $\dfrac{\ln x - 1}{\ln x} = 0$

$\ln x = 1$

$x = e$

Since $x = e$

$f(x) = y = \dfrac{e}{\ln e} = e$ since $\ln e = 1$

So, (e, e) ; $f''(x) = \dfrac{2 - \ln e}{e(\ln e)^3}$

$= \dfrac{2 - 1}{e(1)^3}$

$= \dfrac{1}{e} > 0$

$\therefore$ (e, e) is a min

PTO.

2007
Q2a – 3 marks

Obtain the derivative of each of the following functions:

(*a*) $f(x) = \exp(\sin 2x)$; **3**

Written Solutions

$$f(x) = e^{\sin 2x}$$

$$f'(x) = e^{\sin 2x} \cdot \cos 2x \cdot 2$$

$$= \underline{\underline{2\cos 2x\, e^{\sin 2x}}}$$