Euclidean Algorithm

2013

Q5 – 4 marks

Use the Euclidean algorithm to obtain the greatest common divisor of 1204 and 833, expressing it in the form $1204a + 833b$, where a and b are integers. **4**

Written Solutions

$(1204, 833) = 1204a + 833b$

$1204 = 1 \times 833 + 371$

$833 = 2 \times 371 + 91$

$371 = 4 \times 91 + 7$

$91 = 13 \times 7 + 0$

$\Rightarrow (1204, 833) = 7$

$= 371 - 4 \cdot 91$

$= 371 - 4(833 - 2 \cdot 371)$

$= 9 \cdot 371 - 4 \cdot 833$

$= 9(1204 - 1 \cdot 833) - 4 \cdot 833$

$= 9 \cdot 1204 - 13 \cdot 833$

$\Rightarrow (1204, 833) = 1204(9) + 833(-13)$ i.e. $a = 9$ $b = -13$

2009

Q10 – 4 marks

Use the Euclidean algorithm to obtain the greatest common divisor of 1326 and 14654, expressing it in the form $1326a + 14654b$, where a and b are integers. **4**

Written Solutions

$(1326, 14654) = 1326a + 14654b$

$14654 = 11 \times 1326 + 68$

$1326 = 19 \times 68 + 34$

$68 = 2 \times 34 + 0$

$\Rightarrow (1326, 14654) = 34 = 1326 - 19 \cdot 68$

$= 1326 - 19(14654 - 11 \cdot 1326)$

$= 210 \cdot 1326 - 19 \cdot 14654$

$\therefore (1326, 14654) = 1326(210) + 14654(-19)$ i.e. $a = 210$ $b = -19$

2007

Q7 – 4 marks

Use the Euclidean algorithm to find integers p and q such that $599p + 53q = 1$. **4**

Written Solutions

599 & 53 are coprime.

$$599 = 11 \times 53 + 16$$
$$53 = 3 \times 16 + 5$$
$$16 = 3 \times 5 + 1$$
$$5 = 5 \times 1 + 0$$

$\Rightarrow (599, 53) = 1$

$$\Rightarrow 16 - 3 \cdot 5$$
$$= 16 - 3(53 - 3 \cdot 16)$$
$$= 10(16) - 3 \cdot 53$$
$$= 10(599 - 11 \cdot 53) - 3 \cdot 53$$
$$= 10 \cdot 599 - 113 \cdot 53$$

$\Rightarrow (599, 53) = 599(10) + 53(-113)$ $\therefore$ $p = 10$ $q = -113$