Matrices

2013

Q3 – 5 marks

Matrices A and B are defined by $A=\begin{pmatrix}4 & p\\ -2 & 1\end{pmatrix}$ and $B=\begin{pmatrix}x & -6\\ 1 & 3\end{pmatrix}$.

(a) Find A^2. 1

(b) Find the value of p for which A^2 is singular. 2

(c) Find the values of p and x if $B = 3A'$. 2

Marking Instructions

Expected Answer/s	Max Mark	Additional Guidance
Matrices A and B are defined by $A=\begin{pmatrix}4 & p\\ -2 & 1\end{pmatrix}$ **and** $B=\begin{pmatrix}x & -6\\ 1 & 3\end{pmatrix}$		
(a) Find A^2.	1	
(b) Find the value of p for which A^2 is singular.	2	
(c) Find the values of p and x if $B = 3A'$.	2	
$A^2=\begin{pmatrix}4 & p\\ -2 & 1\end{pmatrix}\begin{pmatrix}4 & p\\ -2 & 1\end{pmatrix}=\begin{pmatrix}16-2p & 4p+p\\ -8-2 & -2p+1\end{pmatrix}$		•[1] Correct answer.[3,1]
$=\begin{pmatrix}16-2p & 5p\\ -10 & 1-2p\end{pmatrix}$		Improved alternative.
A^2 is singular when $\det A^2 = 0$		•[2] Property stated or implied.[4]
$(16-2p)(1-2p)+50p = 0$ $16-34p+4p^2+50p = 0$ $4p^2+16p+16 = 0$ $4(p+2)^2 = 0$ $p = -2$		•[3] Correct value of p.[5,1]
OR A^2 is singular when A is singular, [i.e. when $\det A = 0$]		•[2] Explicitly states property. [not essential, but preferred]
$4+2p = 0$ $p = -2$		•[3] Correct value of p[1]
$A'=\begin{pmatrix}4 & -2\\ p & 1\end{pmatrix}$ $\begin{pmatrix}x & -6\\ 1 & 3\end{pmatrix}=3\begin{pmatrix}4 & -2\\ p & 1\end{pmatrix}$		•[4] A transpose (A^T) correct. Does not have to be explicitly stated.
$\begin{pmatrix}x & -6\\ 1 & 3\end{pmatrix}=\begin{pmatrix}12 & -6\\ 3p & 3\end{pmatrix}$ $\quad x=12, p=\frac{1}{3}$		•[5] Values of p and x correct.[1,2]

2012

Q9 – 4 marks

A non-singular $n \times n$ matrix A satisfies the equation $A + A^{-1} = I$, where I is the $n \times n$ identity matrix. Show that $A^3 = kI$ and state the value of k.

Marking Instructions

Method 1

$A + A^{-1} = I$		
$A^2 + I = A$	**1**	for multiplying by A
Hence $A^2 + I = I - A^{-1}$	**1**	for rearranging $A + A^{-1} = I$
$A^2 = -A^{-1}$	**1**	for subtracting I
$A^3 = -I$, i.e. $k = -1$	**1**	for multiplying by A

Method 2

$A + A^{-1} = I$		
$A = I - A^{-1}$	**1**	for rearranging
$A^2 = I - 2A^{-1} + (A^{-1})^2$	**1**	for squaring
$A^3 = A - 2I + A^{-1}$	**1**	for multiplying by A
$A^3 = (A + A^{-1}) - 2I = I - 2I$		
Hence $A^3 = -I$, i.e. $k = -1$	**1**	

Method 3

$A + A^{-1} = I$		
$A = I - A^{-1}$	**1**	for rearranging
$A^3 = A^2 - A$	**1**	for multiplying by A^2
$A^3 = (A - I) - A$	**1**	using $A^2 = A - I$
$= -I$, i.e. $k = -1$	**1**	

Plus other valid methods.

2012

Q14 – 9 marks

(*a*) Use Gaussian elimination to obtain the solution of the following system of equations in terms of the parameter λ.

$$4x + 6z = 1$$
$$2x - 2y + 4z = -1$$
$$-x + y + \lambda z = 2$$

5

(*b*) Describe what happens when $\lambda = -2$. **1**

(*c*) When $\lambda = -1{\cdot}9$ the solution is $x = -22{\cdot}25$, $y = 8{\cdot}25$, $z = 15$.

Find the solution when $\lambda = -2{\cdot}1$. **2**

Comment on these solutions. **1**

Marking Instructions

(a) $\left\lvert\begin{array}{ccc\|c} 4 & 0 & 6 & 1 \\ 2 & -2 & 4 & -1 \\ -1 & 1 & \lambda & 2 \end{array}\right\rvert$	**1**	for augmented matrix
$\left\lvert\begin{array}{ccc\|c} 4 & 0 & 6 & 1 \\ 0 & 4 & -2 & 3 \\ 0 & 4 & 6+4\lambda & 9 \end{array}\right\rvert$	**1**	
$\left\lvert\begin{array}{ccc\|c} 4 & 0 & 6 & 1 \\ 0 & 4 & -2 & 3 \\ 0 & 0 & 8+4\lambda & 6 \end{array}\right\rvert$	**1**	triangular form needed
$z = \dfrac{6}{8+4\lambda} = \dfrac{3}{2(2+\lambda)}$	**1**	first root
$4y = 3 + 2z \Rightarrow 4y = \dfrac{18+6\lambda}{4+2\lambda}$ $\Rightarrow y = \dfrac{3\lambda+9}{4(2+\lambda)}$ $4x = 1 - 6z \Rightarrow 4x = \dfrac{2\lambda-14}{4+2\lambda}$ $\Rightarrow x = \dfrac{\lambda-7}{4(2+\lambda)}$	**1**	other two roots
(b) When $\lambda = -2$, the final row gives $0 = 6$ which is inconsistent. **There are no solutions**.	**1**	
(c) $\lambda = -2.1$; $x = 22.75$; $y = -6.75$; $z = -15$	**1,1**	1 for first 2 values; 1 for third
Although the values of λ are close, the values of x, y and z are quite different. The system is **ill-conditioned** near $\lambda = -2$.	**1**	

2011

Q4 – 6 marks

(*a*) For what value of λ is $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ -1 & \lambda & 6 \end{pmatrix}$ singular?

(*b*) For $A = \begin{pmatrix} 2 & 2\alpha - \beta & -1 \\ 3\alpha + 2\beta & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}$, obtain values of α and β such that

$$A' = \begin{pmatrix} 2 & -5 & -1 \\ -1 & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}.$$

Marking Instructions

(a) Singular when the determinant is 0.		
$1\det\begin{pmatrix} 0 & 2 \\ \lambda & 6 \end{pmatrix} - 2\det\begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix} + (-1)\det\begin{pmatrix} 3 & 0 \\ -1 & \lambda \end{pmatrix} = 0$	**M1**	
$-2\lambda - 2(18 + 2) - 1(3\lambda - 0) = 0$	**1**	
$-5\lambda - 40 = 0$ when $\lambda = -8$	**1**	
(b) From A, $A' = \begin{pmatrix} 2 & 3\alpha + 2\beta & -1 \\ 2\alpha - \beta & 4 & 3 \\ -1 & 3 & 2 \end{pmatrix}$.	**1**	for transpose
Hence $2\alpha - \beta = -1$ and $3\alpha + 2\beta = -5$.	**1**	
$4\alpha - 2\beta = -2$		
$3\alpha + 2\beta = -5$		
$7\alpha = -7 \Rightarrow \alpha = -1$ and $\beta = -1$.	**1**	for both values

2010

Q4 – 4 marks

Obtain the 2×2 matrix M associated with an enlargement, scale factor 2, followed by a clockwise rotation of $60°$ about the origin.

Marking Instructions

The matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ gives an enlargement, scale factor 2.	**1**	correct matrix
The matrix $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ gives a clockwise rotation of 60° about the origin.	**1**	correct matrix
$M = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$	**1**	correct order
$= \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$.	**1**	

2010

Q14 – 10 marks

Use Gaussian elimination to show that the set of equations

$$\begin{aligned} x - y + z &= 1 \\ x + y + 2z &= 0 \\ 2x - y + az &= 2 \end{aligned}$$

has a unique solution when $a \neq 2{\cdot}5$. **5**

Explain what happens when $a = 2{\cdot}5$. **1**

Obtain the solution when $a = 3$. **1**

Given $A = \begin{pmatrix} 5 & 2 & -3 \\ 1 & 1 & -1 \\ -3 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, calculate AB. **1**

Hence, or otherwise, state the relationship between A and the matrix

$$C = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 2 & -1 & 3 \end{pmatrix}.$$ **2**

Marking Instructions

$\left(\begin{array}{ccc\|c} 1 & -1 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 2 & -1 & a & 2 \end{array}\right)$	**1**	for a structured approach
$\left(\begin{array}{ccc\|c} 1 & -1 & 1 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 1 & a-2 & 0 \end{array}\right)$	**1**	
$\left(\begin{array}{ccc\|c} 1 & -1 & 1 & 1 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 2a-5 & 1 \end{array}\right)$	**1**	for triangular form
$z = \dfrac{\mathbf{1}}{\mathbf{2a - 5}}$;	**1**	one correct variable
$2y + \dfrac{1}{2a-5} = -1 \Rightarrow 2y = \dfrac{-2a+5-1}{2a-5}$ $\Rightarrow y = \dfrac{\mathbf{2-a}}{\mathbf{2a-5}}$; $x - \dfrac{2-a}{2a-5} + \dfrac{1}{2a-5} = 1$ $\Rightarrow x = \dfrac{2a-5}{2a-5} + \dfrac{1-a}{2a-5} = \dfrac{\mathbf{a-4}}{\mathbf{2a-5}}$.	**1**	for the two other variables {*other justifications for uniqueness are possible*}
which exist when $2a - 5 \neq 0$.		
From the third row of the final tableau, when $a = 2{\cdot}5$, there are no solutions	**1**	
When $a = 3$, $x = -1$, $y = -1$, $z = 1$.	**1**	
$AB = \begin{pmatrix} 5 & 2 & -3 \\ 1 & 1 & -1 \\ -3 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$	**1**	
From above, we have $C\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and also $A\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ which suggests $AC = I$ and this can be verified directly. Hence A is the inverse of C (or vice versa).	**2**	A candidate who obtains $AC = I$ directly may be awarded full marks.

2009

Q2 – 5 marks

Given the matrix $A = \begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix}$.

(a)	Find A^{-1} in terms of t when A is non-singular.	3
(b)	Write down the value of t such that A is singular.	1
(c)	Given that the transpose of A is $\begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix}$, find t.	1

Marking Instructions

(a)

$$\det\begin{pmatrix} t+4 & 3t \\ 3 & 5 \end{pmatrix} = 5(t+4) - 9t \qquad \mathbf{1}$$

$$= 20 - 4t$$

$$A^{-1} = \frac{1}{20-4t}\begin{pmatrix} 5 & -3t \\ -3 & t+4 \end{pmatrix} \qquad \mathbf{1,1}$$

(b)

$$20 - 4t = 0 \Rightarrow t = 5 \qquad \mathbf{1}$$

(c)

$$\begin{pmatrix} t+4 & 3 \\ 3t & 5 \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 6 & 5 \end{pmatrix} \Rightarrow t = 2 \qquad \mathbf{1}$$

2009

Q16(a) – 5 marks

Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} x + y - z &= 6 \\ 2x - 3y + 2z &= 2 \\ -5x + 2y - 4z &= 1. \end{aligned}$$

Marking Instructions

$$\begin{aligned} x + y - z &= 6 \\ 2x - 3y + 2z &= 2 \\ -5x + 2y - 4z &= 1 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 2 & -3 & 2 & 2 \\ -5 & 2 & -4 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -5 & 4 & -10 \\ 0 & 7 & -9 & 31 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -5 & 4 & -10 \\ 0 & 0 & -\frac{17}{5} & 17 \end{array}\right)$$

1,1,1

$$z = 17 \div \left(\frac{-17}{5}\right) = -5 \qquad \mathbf{1}$$

$$-5y - 20 = -10 \Rightarrow y = -2$$

$$x - 2 + 5 = 6 \Rightarrow x = 3 \qquad \mathbf{1}$$

2008

Q6 – 5 marks

Let the matrix $A = \begin{pmatrix} 1 & x \\ x & 4 \end{pmatrix}$.

(*a*) Obtain the value(s) of x for which A is singular.

(*b*) When $x = 2$, show that $A^2 = pA$ for some constant p.
Determine the value of q such that $A^4 = qA$.

Marking Instructions

(a)

$$\det\begin{pmatrix} 1 & x \\ x & 4 \end{pmatrix} = 4 - x^2 \qquad \mathbf{1}$$

A matrix is singular when its determinant is 0, hence $x = \pm 2$. **1**

(b) When $x = 2$, $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ 10 & 20 \end{pmatrix} = 5A. \qquad \mathbf{1}$$

$$A^4 = (A^2)^2 = (5A)^2 = 25A^2 = 125A. \qquad \mathbf{2E1}$$

Evaluating $A^4 = \begin{pmatrix} 125 & 250 \\ 250 & 500 \end{pmatrix} = 125A$ was accepted.

2007

Q5 – 5 marks

Matrices A and B are defined by

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}, \qquad B = \begin{pmatrix} x+2 & x-2 & x+3 \\ -4 & 4 & 2 \\ 2 & -2 & 3 \end{pmatrix}.$$

(*a*) Find the product AB.

(*b*) Obtain the determinants of A and of AB.

Hence, or otherwise, obtain an expression for det B.

Marking Instructions

(a)

$$AB = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} x+2 & x-2 & x+3 \\ -4 & 4 & 2 \\ 2 & -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} x & x & x \\ -6 & 6 & -1 \\ 0 & 0 & 8 \end{pmatrix} \qquad \textbf{2E1}$$

(b)

$$\det A = 1 \times (2 + 1) - 0 - 1 \times 0 = 3 \qquad \textbf{1}$$

$$\det AB = x(48 - 0) - x(-48 - 0) + x(0 - 0) = 96x \qquad \textbf{1}$$

Since $\det AB = \det A \det B$

$$\det B = \frac{\det AB}{\det A} = \frac{96x}{3} = 32x \qquad \textbf{1}$$

2005

Q6 – 6 marks

Use Gaussian elimination to solve the system of equations below when $\lambda \neq 2$:

$$\begin{aligned} x + y + 2z &= 1 \\ 2x + \lambda y + z &= 0 \\ 3x + 3y + 9z &= 5. \end{aligned}$$

Explain what happens when $\lambda = 2$.

Marking Instructions

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & \lambda & 1 & 0 \\ 3 & 3 & 9 & 5 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & \lambda - 2 & -3 & -2 \\ 0 & 0 & 3 & 2 \end{array}\right) \qquad \textbf{2E1}$$

$z = \frac{2}{3}$; **1**

$(\lambda - 2)y - 2 = -2 \Rightarrow y = 0;\ x = 1 - 0 - \frac{4}{3} = -\frac{1}{3}$. **1**

When $\lambda = 2$, the second and third rows of the second matrix are the same, **1(†)**
so there is an infinite number of solutions. **1**

(†) Use of ‘redundant’ is worth a mark.

Interpretation in geometrical terms can be given both the marks.

2005

Q7 – 6 marks

Given the matrix $A = \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$, show that $A^2 + A = kI$ for some constant k, where I is the 3×3 unit matrix.

Obtain the values of p and q for which $A^{-1} = pA + qI$.

Marking Instructions

$A = \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} \Rightarrow A^2 = \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}\begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & -4 & -2 \\ -1 & 2 & -1 \\ 1 & 2 & 5 \end{pmatrix}$	**2E1**
$A^2 + A = \begin{pmatrix} 2 & -4 & -2 \\ -1 & 2 & -1 \\ 1 & 2 & 5 \end{pmatrix} + \begin{pmatrix} 0 & 4 & 2 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{pmatrix}$	**1**
$= \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = 2I$	**1**
$A^{-1}(A^2 + A) = 2A^{-1}$	**1**
$2A^{-1} = A + I$	
$A^{-1} = \frac{1}{2}A + \frac{1}{2}I$	**1**

2004

Q6 – 5 marks

Write down the 2×2 matrix M_1 associated with an anti-clockwise rotation of $\frac{\pi}{2}$ radians about the origin.

Write down the matrix M_2 associated with reflection in the x-axis.

Evaluate $M_2 M_1$ and describe geometrically the effect of the transformation represented by $M_2 M_1$.

Marking Instructions

$M_1 = \begin{pmatrix} \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} \\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	**2**
$M_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	**1**
$M_2M_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	**1**
The transformation represented by M_2M_1 is reflection in $y = -x$.	**1**

2003

A6 – 6 marks

Use elementary row operations to reduce the following system of equations to upper triangular form

$$\begin{aligned} x + y + 3z &= 1 \\ 3x + ay + z &= 1 \\ x + y + z &= -1. \end{aligned}$$

Hence express x, y and z in terms of the parameter a.

Explain what happens when $a = 3$.

Marking Instructions

$$\begin{aligned} x + y + 3z &= 1 \\ 3x + ay + z &= 1 \\ x + y + z &= -1. \end{aligned}$$

Hence

$$\begin{aligned} x + y + 3z &= 1 \\ (a-3)y - 8z &= -2 \\ -2z &= -2 \end{aligned}$$

2†

When $a \neq 3$, we can solve to give a unique solution.

$$z = 1; \quad y = \frac{6}{a-3}; \quad x = -2 + \frac{6}{3-a}.$$

2E1

When $a = 3$, we get $z = \frac{1}{4}$ from the second equation but $z = 1$‡ from the third, i.e. inconsistent§ . **2**

2002

A1 – 5 marks

Use Gaussian elimination to solve the following system of equations

$$\begin{aligned} x + y + 3z &= 2 \\ 2x + y + z &= 2 \\ 3x + 2y + 5z &= 5. \end{aligned}$$

Marking Instructions

$$\begin{matrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 2 \\ 3 & 2 & 5 & 5 \end{matrix} \Rightarrow \begin{matrix} 1 & 1 & 3 & 2 \\ & -1 & -5 & -2 \\ & -1 & -4 & -1 \end{matrix}$$

$$\Rightarrow \begin{matrix} 1 & 1 & 3 & 2 \\ & -1 & -5 & -2 \\ & & -1 & -1 \end{matrix}$$

$z = 1; \quad y = -3; \quad x = 2$

Second row 1 mark Third row 1 mark
Third row 1 mark
Values 2E1. (available whatever method used above) Total 5

2002

B4 – 4 marks

Write down the 2×2 matrix A representing a reflection in the x-axis and the 2×2 matrix B representing an anti-clockwise rotation of $30°$ about the origin.

Hence show that the image of a point (x, y) under the transformation A followed by the transformation B is $\left(\dfrac{kx+y}{2}, \dfrac{x-ky}{2}\right)$, stating the value of k.

Marking Instructions

$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	1 for A
$B = \begin{pmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}$	1 for B
$BA\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$	1 method for tackling a compostion
$= \frac{1}{2}\begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \sqrt{3}x + y \\ x - \sqrt{3}y \end{pmatrix}$	
i.e. $(x, y) \to \frac{1}{2}(\sqrt{3}x + y, x - \sqrt{3}y)$	
so $k = \sqrt{3}$.	1 for value of k Total 4