

Partial Fractions

2012

Q15(a) – 4 marks

Express $\frac{1}{(x-1)(x+2)^2}$ in partial fractions.

Written Solutions

☁ Type 2 (as per notes)

$$\frac{1}{(x-1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

x both sides
by $(x-1)(x+2)^2$

$$1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$$\text{let } x = 1$$

$$\text{let } x = -2$$

$$\text{let } x = 0$$

$$1 = A(3)^2 \Rightarrow 1 = 9A \Rightarrow A = \frac{1}{9}$$

$$1 = -3C \Rightarrow C = -\frac{1}{3}$$

$$1 = A(2)^2 + B(-1)(2) + C(-1)$$

$$1 = 4A - 2B - C$$

$$1 = 4\left(\frac{1}{9}\right) - 2B - \left(-\frac{1}{3}\right)$$

$$1 = \frac{7}{9} - 2B$$

$$B = -\frac{1}{9}$$

$$\text{So, } \underline{\underline{\frac{1}{(x-1)(x+2)^2} = \frac{1}{9(x-1)} - \frac{1}{9(x+2)} - \frac{1}{3(x+2)^2}}}$$

2011

Q1 (a bit of) - 3 marks

Express $\frac{13-x}{x^2+4x-5}$ in partial fractions

Written Solutions

☺ Type 1 (as per notes)

$$\frac{13-x}{x^2+4x-5} = \frac{13-x}{(x-1)(x+5)} = \frac{A}{x-1} + \frac{B}{x+5}$$

*x both sides
by $(x-1)(x+5)$*

$$13-x = A(x+5) + B(x-1)$$

let $x = 1$

$$\begin{aligned} 13-1 &= A(1+5) \\ 12 &= 6A \\ (\div 6) \quad A &= 2 \end{aligned}$$

let $x = -5$

$$\begin{aligned} 13-(-5) &= B(-5-1) \\ 18 &= -6B \\ (\div -6) \quad B &= -3 \end{aligned}$$

So,

$$\underline{\underline{\frac{13-x}{(x-1)(x+5)} = \frac{2}{x-1} - \frac{3}{x+5}}}$$

2010

Q7 (a bit of) - 4 marks

Express $\frac{3x+5}{(x+1)(x+2)(x+3)}$ in partial fractions.

Written Solutions

o ☁ Type 1 (as per notes)

$$\frac{3x+5}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$$

x both sides by
 $(x+1)(x+2)(x+3)$

$$3x+5 = A(x+2)(x+3) + B(x+1)(x+3) + C(x+1)(x+2)$$

let $x = -1$

$$3(-1)+5 = A(-1+2)(-1+3)$$

$$2 = 2A$$

$$A = 1$$

let $x = -2$

$$3(-2)+5 = B(-2+1)(-2+3)$$

$$-1 = -B$$

$$B = 1$$

let $x = -3$

$$3(-3)+5 = C(-3+1)(-3+2)$$

$$-4 = 2C$$

$$C = -2$$

$$\text{So, } \frac{3x+5}{(x+1)(x+2)(x+3)} = \frac{1}{x+1} + \frac{1}{x+2} - \frac{2}{x+3}$$

2009

Q14 (a bit of) - 4 marks

Express $\frac{x^2+6x-4}{(x+2)^2(x-4)}$ in partial fractions.

Written Solutions

oo ☺ Type 2 (as per notes)

$$\frac{x^2+6x-4}{(x+2)^2(x-4)} = \frac{A}{x-4} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

* both sides by
 $(x+2)^2(x-4)$

$$x^2+6x-4 = A(x+2)^2 + B(x+2)(x-4) + C(x-4)$$

let $x = 4$

$$36 = A(6)^2$$

$$36 = 36A$$

$$A = 1$$

let $x = -2$

$$-12 = C(-6)$$

$$C = 2$$

let $x = 0$

$$-4 = A(2)^2 + B(2)(-4) + C(-4)$$

$$-4 = 4A - 8B - 4C$$

$$-4 = 4(1) - 8B - 4(2)$$

$$B = 0$$

$$\text{So, } \underline{\underline{\frac{x^2+6x-4}{(x+2)^2(x-4)} = \frac{1}{x-4} + \frac{2}{(x+2)^2}}}$$

2005

Q13 (a bit of) - 4 marks

Express $\frac{1}{x^3+x}$ in partial fractions.

Written Solutions

☁ Type 3 (as per notes)

$$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)}$$

Check irreducible? x^2+1

$$b^2-4ac = 0^2-4(1)(1) = -4 \Rightarrow \text{no real roots}$$

$$\text{So, } \frac{1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

\times both sides by $x(x^2+1)$

$$1 = A(x^2+1) + (Bx+C)x$$

let $x=0$

$$1 = A(0^2+1)$$
$$A = 1$$

let $x=-1$

$$1 = A((-1)^2+1) + (B(-1)+C)(-1)$$
$$1 = 2A + B - C$$

Using $A=1$ (above)

$$1 = 2 + B - C$$
$$-1 = B - C$$

let $x=1$

$$1 = A(1^2+1) + (B(1)+C)(1)$$
$$1 = 2A + B + C$$

Using $A=1$ (above)

$$1 = 2 + B + C$$
$$-1 = B + C$$

Sim. Eqⁿ

$$B + C = -1 \quad \textcircled{1}$$

$$B - C = -1 \quad \textcircled{2}$$

$$\dots B = -1 \text{ and } C = 0$$

(intuition also tells us this !!)

$$\text{So, } \frac{1}{x^3+x} = \frac{1}{x} - \frac{x}{x^2+1}$$

2004

Q5 (a bit of) - 2 marks

Express $\frac{1}{x^2-x-6}$ in partial fractions.

Written Solutions

60 ☁ Type 1 (as per notes)

$$\frac{1}{x^2-x-6} = \frac{1}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

* both sides
by $(x-3)(x+2)$

$$1 = A(x+2) + B(x-3)$$

$$\text{let } x = 3 \quad 1 = A(3+2)$$

$$A = \frac{1}{5}$$

$$\text{let } x = -2 \quad 1 = B(-2-3)$$

$$B = -\frac{1}{5}$$

$$\text{So, } \underline{\underline{\frac{1}{x^2-x-6} = \frac{1}{5(x-3)} - \frac{1}{5(x+2)}}}}$$