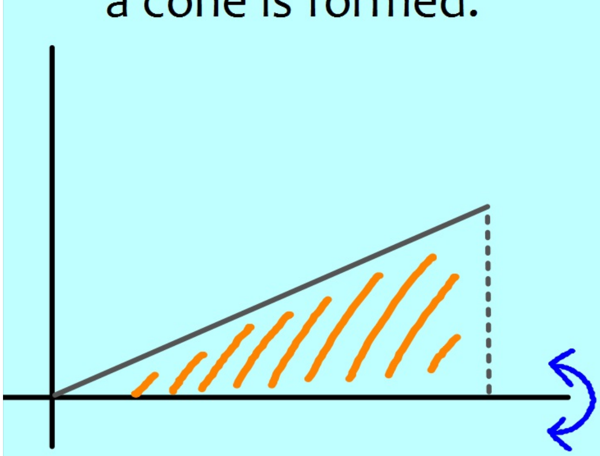
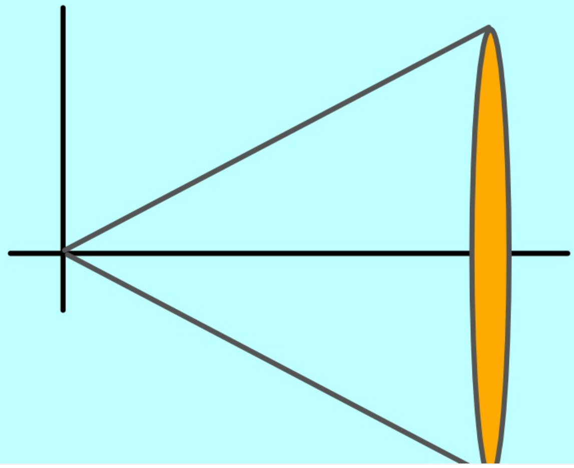
Volume of a Solid of Revolution

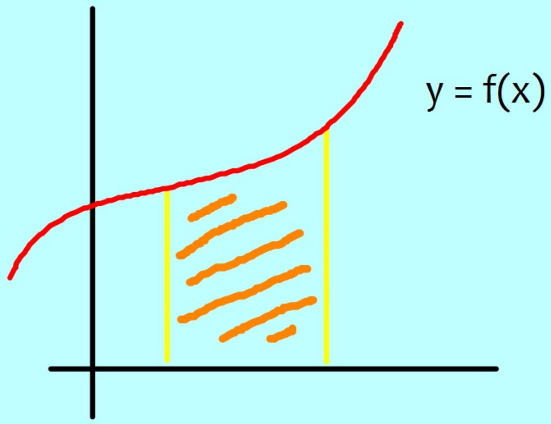
A solid of revolution is formed when a curve is rotated about the x-axis or y-axis.

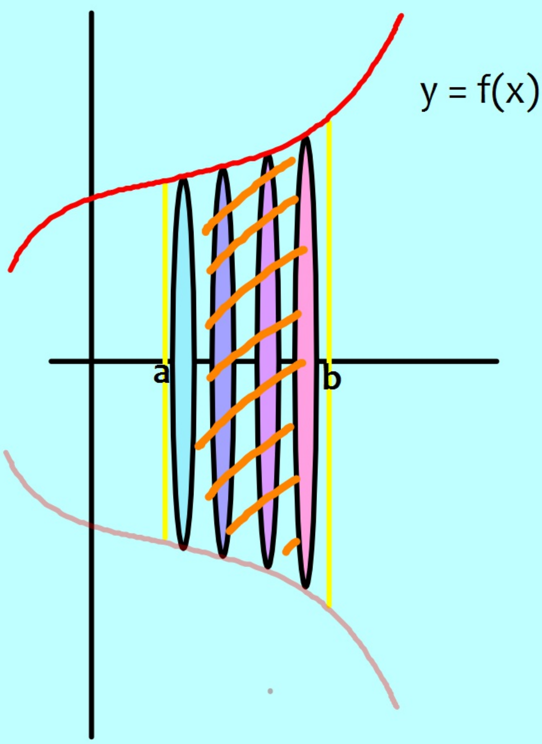
e.g. when the line y = kx is rotated about the x-axis,
a cone is formed.





Rotation about the x-axis

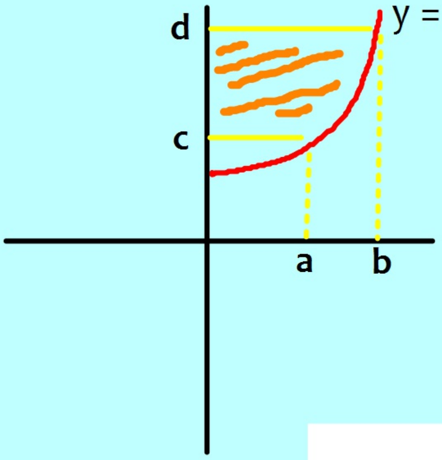


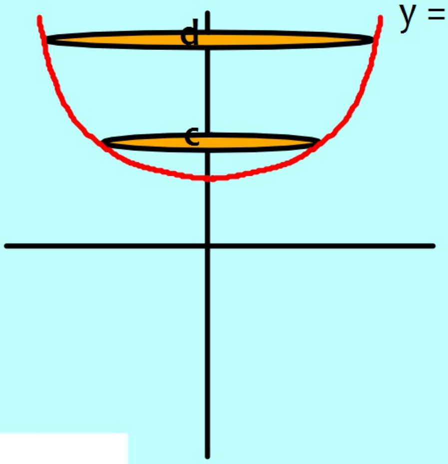


$$V = \pi \int_a^b y^2 dx$$

$$V = \pi \int_a^b f(x)^2 dx$$

Rotation about the y-axis

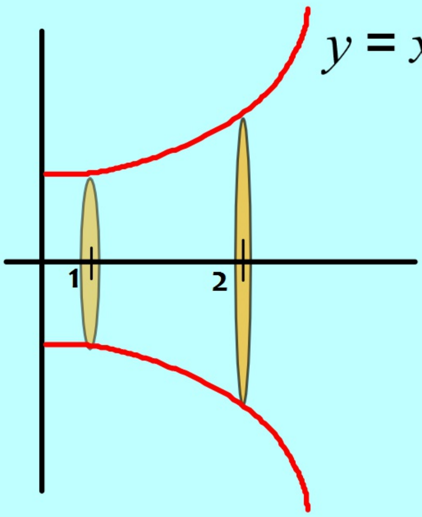




$$V = \pi \int_c^d x^2 dy$$

$$V = \pi \int_c^d f(y)^2 dy$$

Example 1



$$y = x^2 + 1$$

$$V = \pi \int_a^b y^2 dx$$

$$V = \pi \int_1^2 (x^2+1)^2 dx$$

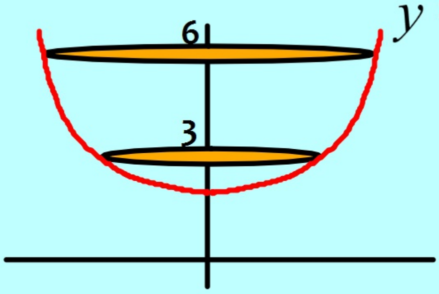
$$V = \pi \int_1^2 x^4 + 2x^2 + 1\, dx$$

$$V = \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} + x\right]_1^2$$

$$V = \pi \left[\left(\frac{32}{5} + \frac{16}{3} + 2\right) - \left(\frac{1}{5} + \frac{2}{3} + 1\right)\right]$$

$$V = 11\frac{13}{15}\pi\ unit^3$$

Example 2



$y = x^2 + 1$

$$V = \pi \int_c^d x^2 dy$$

$y = x^2 + 1 \qquad \therefore x^2 = y - 1$

$$V = \pi \int_3^6 y - 1\, dy$$

$$V = \pi \left[\frac{y^2}{2} - y\right]_3^6$$

$$V = \pi[(18 - 6) - (4.5 - 3)]$$

$$V = \underline{\underline{10.5\pi\ unit^3}}$$

2007

Q10 – 6 marks

Use the substitution $u = 1 + x^2$ to obtain $\int_0^1 \frac{x^3}{(1+x^2)^4}\, dx$. **5**

A solid is formed by rotating the curve $y = \frac{x^{3/2}}{(1+x^2)^2}$ between $x = 0$ and $x = 1$ through $360°$ about the x-axis. Write down the volume of this solid. **1**

2010

Q15 – 10 marks

A new board game has been invented and the symmetrical design on the board is made from four identical "petal" shapes. One of these petals is the region enclosed between the curves $y = x^2$ and $y^2 = 8x$ as shown shaded in diagram 1 below.

Calculate the area of the complete design, as shown in diagram 2. **5**

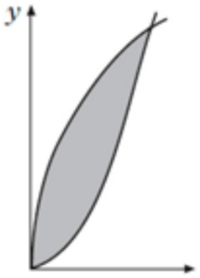


Diagram 1

Diagram 2

The counter used in the game is formed by rotating the shaded area shown in diagram 1 above, through $360°$ about the y-axis. Find the volume of plastic required to make one counter. **5**