Differentiation

Q1 $y = 3x^{-2} + 2x^{3/2}$

$$\frac{dy}{dx} = -6x^{-3} + 3x^{\frac{1}{2}}$$

C

Q2 $f(x) = \sqrt{x} + \frac{2}{x^2}$

$$= x^{\frac{1}{2}} + 2x^{-2}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 4x^{-3}$$

$$= \frac{1}{2\sqrt{x}} - \frac{4}{x^3}$$

$$f'(4) = \frac{1}{2\times\sqrt{4}} - \frac{4}{4^3}$$

$$= \frac{1}{2\times 2} - \frac{1}{4^2}$$

$$= \frac{1}{4} - \frac{1}{16}$$

$$= \frac{4}{16} - \frac{1}{16}$$

$$= \frac{3}{16}$$

Q3 $s(t) = t^2 - 5t + 8$

$s'(t) = 2t - 5$

$s'(3) = 2(3) - 5$

$= \underline{1}$

B

Q4 $y = 3\sin x + \cos 2x$

$\frac{dy}{dx} = 3\cos x - 2\sin 2x$

Q5 $f(x) = 4\sin 3x$

$f'(x) = 4\cos 3x \times 3$

$= 12\cos 3x$

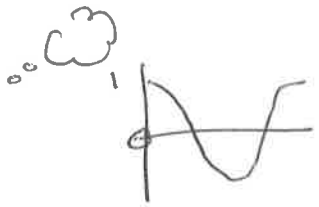
$f'(0) = 12\cos(3\times 0)$

$= 12\cos 0$

$= 12\times 1$

$= \underline{12}$

C



Q6 $y = 3\cos^4 x$

$y = 3(\cos x)^4$

$\frac{dy}{dx} = 12(\cos x)^3 \times -\sin x$

$= -12\cos^3 x \sin x$ C

Q7 $y = 5x^3 - 12x$

$\frac{dy}{dx} = 15x^2 - 12$

When $x = 1$, $m = 15(1)^2 - 12$

$= 15 - 12$

$= 3$ C

Q8 $y = x^3 - 2x$

$\frac{dy}{dx} = 3x^2 - 2$

When $x = 2$, $m = 3(2)^2 - 2$

$= 12 - 2$

$= 10$

Q9

GRADIENT

$$y = \frac{24}{\sqrt{x}}$$

$$= \frac{24}{x^{\frac{1}{2}}}$$

$$= 24x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -12x^{-\frac{3}{2}}$$

$$= \frac{-12}{x^{3/2}}$$

$$= -\frac{12}{\sqrt{x}^3} \rightarrow \text{When } x = 4, \; m = -\frac{12}{\sqrt{4}^3}$$

$$= -\frac{12}{2^3}$$

$$= -\frac{12}{8}$$

$$= -\frac{3}{2}$$

POINT

$$y = \frac{24}{\sqrt{4}}$$

$$= \frac{24}{2}$$

$$= 12 \quad \text{i.e. } (4, 12)$$

Equation

$m = -\frac{3}{2}$

$(4, 12)$

$$y - b = m(x - a)$$

$$y - 12 = -\frac{3}{2}(x - 4)$$

COULD CONTINUE AS FOLLOWS:

$(\times 2)$ $2y - 24 = -3(x - 4)$

$2y - 24 = -3x + 12$

$(+3x)$ $3x + 2y - 24 = 12$

(-12) $3x + 2y - 36 = 0$

Q10

a) $y = x^3 + 2x^2 - 3x + 2$

GRADIENT $\frac{dy}{dx} = 3x^2 + 4x - 3$

When $x = 1$, $m = 3(1)^2 + 4(1) - 3$

$= 3 + 4 - 3$

$= \underline{4}$

POINT When $x = 1$, $y = x^3 + 2x^2 - 3x + 2$

$= 1^3 + 2(1)^2 - 3(1) + 2$

$= 1 + 2 - 3 + 2$

$= 2$ i.e. $(1, 2)$

EQUATION $y - b = m(x - a)$

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4$

$y = \underline{4x - 2}$

b) $x^2 + y^2 - 12x - 10y + 44 = 0$

$x^2 + (4x - 2)^2 - 12x - 10(4x - 2) + 44 = 0$

$x^2 + 16x^2 - 16x + 4 - 12x - 40x + 20 + 44 = 0$

$17x^2 - 68x + 68 = 0$

$(\div 17)$ $x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

..C?

$(4x - 2)(4x - 2)$

$16x^2 - 8x - 8x + 4$

$16x^2 - 16x + 4$

Since only one solution the line is a tangent.

$x - 2 = 0$

$x = 2$ i.e. $(2, ?)$

When $x = 2$ $y = 4(2) - 2$

$= 6$ i.e. $\underline{(2, 6)}$ is the point of contact.