



**KEEP
CALM
AND
REVISE**

Introduction

This booklet is designed to help you prepare for your upcoming Maths prelim. It contains a selection of questions from each of the following topics,

- The Straight Line
- Vectors
- The Circle
- Recurrence Relationships (Sequences)
- Functions and Graphs
- Differentiation
- Optimisation (Max/Min Problems)
- Integration
- Finding the Area Under a Curve
- Quadratics Functions
- Polynomials

You can expect each of these topics to appear in your prelim, so it would be advisable to try as many of the practice questions as you can and if you require any extra help or advice please see your teacher at the earliest opportunity.



The use of the following calculator symbol indicates that a calculator may be used.

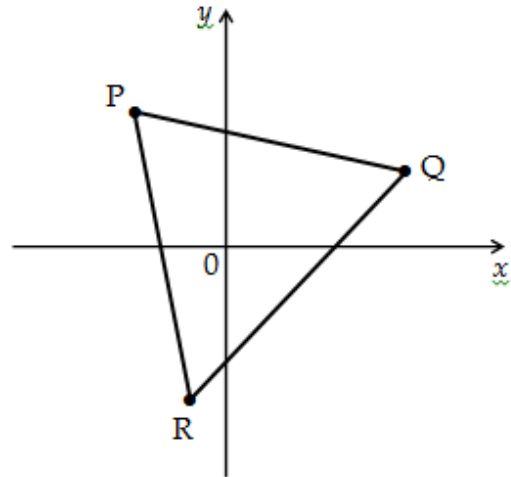
All other questions should be considered non-calculator.

Additional materials for revision can be found on the Maths Department website at

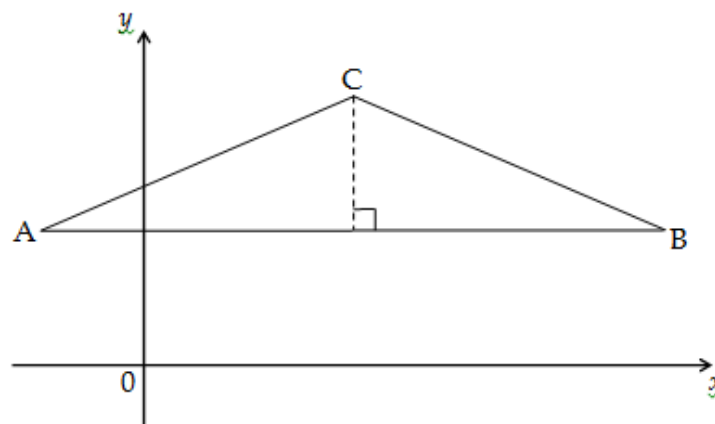
The Straight Line

MARKS

1. Triangle PQR has vertices $P(-3, 5)$, $Q(7, 3)$ and $R(-1, -5)$, as shown.

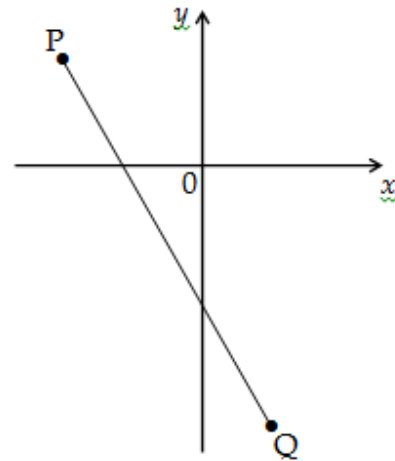


- (a) Find the equation of the median RM. 3
- (b) Find the equation of the altitude AP. 3
- (c) Find the coordinates of the point of intersection of RM and AP. 2
2. $A(-2, 4)$, $B(10, 4)$ and $C(4, 8)$ are the vertices of triangle ABC shown in the diagram.



- (a) Write down the equation of the altitude from C. 1
- (b) Find the equation of the perpendicular bisector of BC. 4
- (c) Find the point of intersection of the lines found in (a) and (b). 2

3. A line joins the points $P(-4, 3)$ and $Q(2, -7)$.
Find the equation of the perpendicular Bisector of PQ .



4

4. The diagram opposite shows a rhombus $ABCD$.

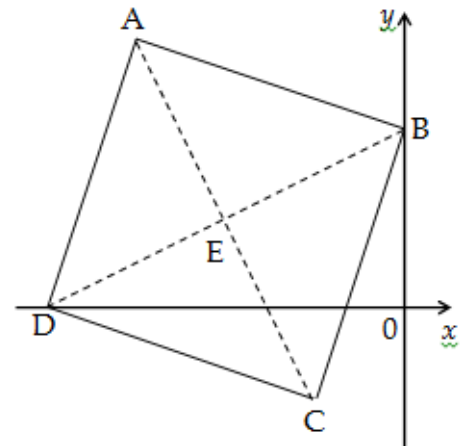


AC and BD are diagonals of the Rhombus.

Diagonal AC has equation $2x + y + 3 = 0$.

D is point with coordinates $(-4, 0)$.

E is the point of intersection of the diagonals.

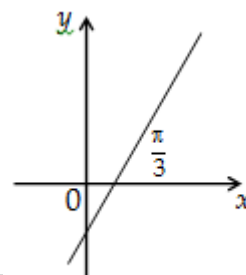


- (a) Find the equation of diagonal BD .
(b) Hence find the coordinates of E .

4

2

5. The angle between the line shown in the diagram and the x -axis is $\frac{\pi}{3}$.
What is the gradient of the line?



2

Vectors

1. The vectors p and q with components $p = \begin{pmatrix} 1 \\ k \\ 2 \end{pmatrix}$ and $q = \begin{pmatrix} k \\ -3 \\ -2 \end{pmatrix}$ are perpendicular.

What is the value of k ?

2

2. K is the point $(3, -2, 3)$, L $(5, 0, 7)$ and M $(7, -3, -1)$.



(a) Write down the components of \overline{KL} and \overline{KM} .

2

(b) Calculate the size of angle LKM.

5

3. Vector t has components $\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$. u is a unit vector such that $u = kt$, where $k > 0$.

Find the value of k .

2

4. A is the point with coordinates $(1, -1, 2)$, B $(3, 0, 3)$ and C $(-2, 3, 4)$.



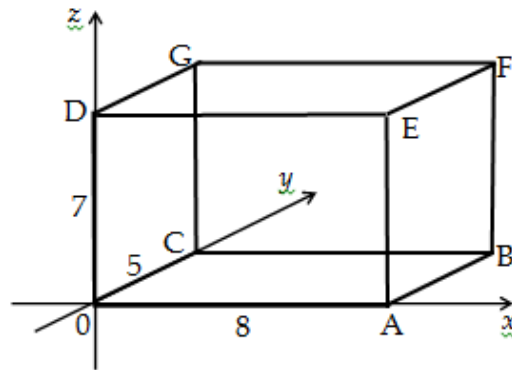
(a) Express \overline{AB} and \overline{AC} in component form.

2

(b) Find the size of angle BAC.

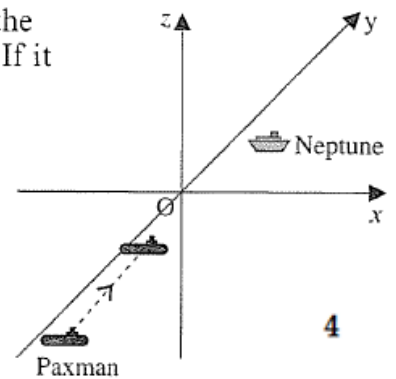
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5. OABC,DEFG is a rectangular prism as show.



OA is 8 units long, OC is 5 units and OD is 7 units.

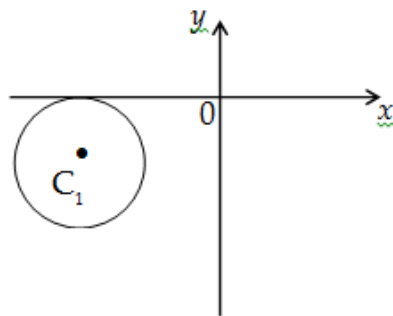
- (a) Write down the coordinates of B and G. 2
- (b) Calculate the size of angle BEG. 6
6. At 1300 hours the submarine Paxman is located by sonar at the point $(-5, -2, 9)$. One hour later it is located at $(-1, 0, -3)$. If it continues on the same course will it hit the fishing vessel, Neptune, located at point $(1, 1, 0)$?



4

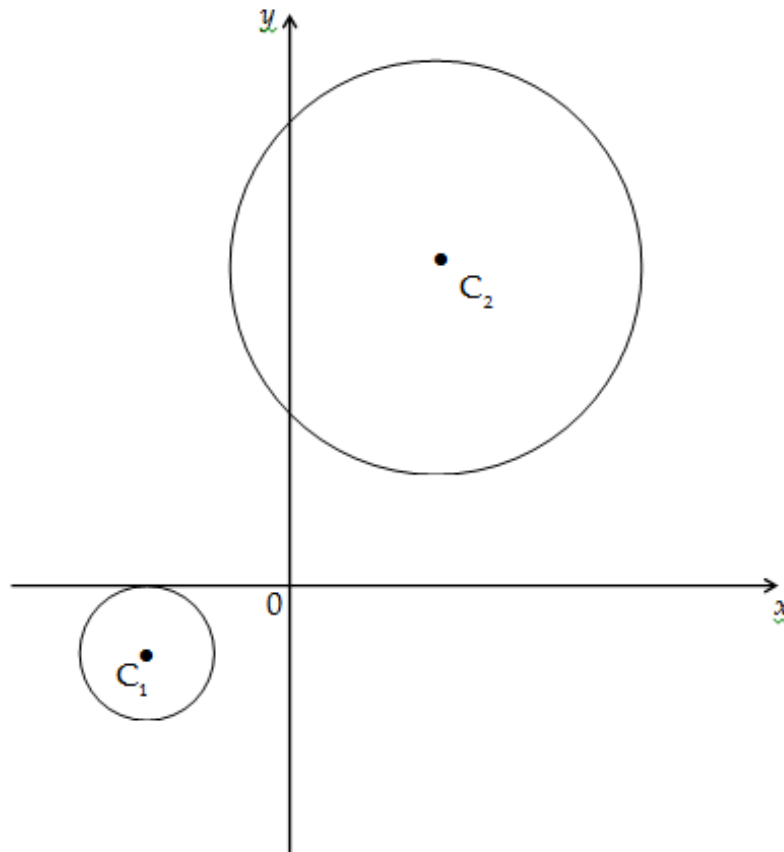
The Circle

1. A circle with equation $x^2 + y^2 + 6x + 2y + 9 = 0$ has centre C_1 .



- (a) Write down the coordinates of the centre C_1 and find the length of the Radius of this circle. 2

A second circle with equation $(x - 3)^2 + (y - 7)^2 = 36$ has centre C_2 .



- 7 | (b) (i) Find the distance between the centres C_1 and C_2 .
(ii) Hence find the minimum distance between the circumferences of the two circles. 4

2. A circle with centre $(-2, 1)$ passes through the point $(5, -2)$.

What is the equation of the circle?

2

3. A circle, centre C , has equation $x^2 + y^2 - 4x - 2y - 20 = 0$.



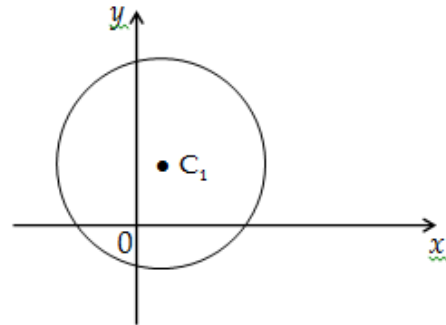
- (a) Find the centre C and radius of this circle. 2
- (b) (i) Show that the point $P(5, -3)$ lies on the circumference of the circle. 4
- (ii) Find the equation of radius CP . 4
- (c) Find the equation of the chord which passes through $(7, 1)$ and is perpendicular to radius CP . 3

4. A circle with centre C_1 has



equation $x^2 + y^2 - 2x - 6y - 15 = 0$.

- (a) Write down the coordinates of the centre and calculate the length of the radius of this circle.

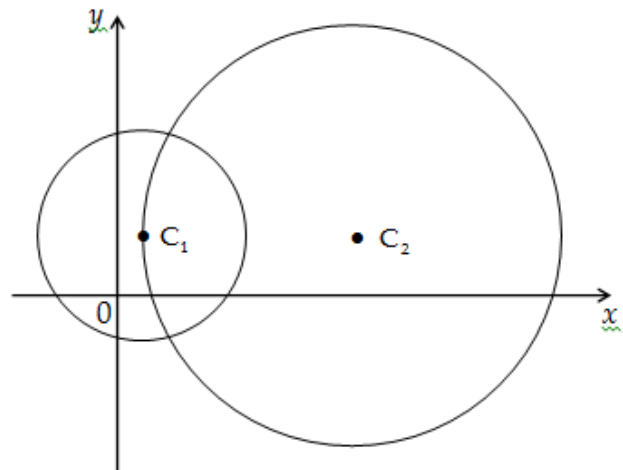


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A second circle with centre C_2 has a diameter twice that of the circle with centre C_1 .

C_1 lies on the circumference of this second circle.

The line joining C_1 and C_2 is parallel to the x -axis.



- (b) Find the equation of the circle with centre C_2 .

4

5. Show that the line with equation $y = 2x + 10$ is a tangent to the circle with equation $x^2 + y^2 - 2x - 4y - 15 = 0$ and find the coordinates of the point of contact of the tangent and circle.

6

6. A circle has the following properties:



- The x -axis and the line $y = 20$ are tangents to the circle.
- The circle passes through the points $(0, 2)$ and $(0, 18)$.
- The centre lies in the first quadrant.

Find the equation of this circle.

6

Recurrence Relationships (Sequences)

1. (a) A sequence is defined by the recurrence relation $u_{n+1} = 0.4u_n + 6$, $u_0 = 0$.
Determine the values of u_1 , u_2 and u_3 . 2
- (b) Why does this sequence have a limit as $n \rightarrow \infty$? 1
- (c) A second sequence, generated by $v_{n+1} = pv_n + 4$, has the same limit as the sequence in (a).
Find the value of p . 3
2. Two sequences are generated by the recurrence relations
- $$u_{n+1} = 0.4u_n + 8 \cdot 4$$
- $$v_{n+1} = kv_n + 2$$
- The two sequences approach the same limit as $n \rightarrow \infty$.
- (a) Evaluate this limit. 2
- (b) Hence determine the value of k . 2
3. A sequence is defined by recurrence relation $u_{n+1} = ku_n - 6$, $u_0 = 0$.
- (a) Given that $u_2 = -8$, find the value of k . 2
- (b) (i) Why does this sequence tend to a limit as $n \rightarrow \infty$?
(ii) Find the value of this limit. 3

4. A new '24 hour anti-biotic' is being tested on a patient in hospital.



It is known, that over a 24 hour period, the amount of anti-biotic remaining in the bloodstream is reduced by 80%.

On the first day of the trial, an initial 250 mg dose is given to a patient at 7 a.m.

(a) After 24 hours and just prior to the second dose being given, how much anti-biotic remains in the patient's bloodstream? 1

The patient is then given a further 250 mg dose at 7 a.m. and at this time each subsequent morning thereafter.

(b) A recurrence relation of the form $u_{n+1} = au_n + b$ can be used to model this course of treatment.

Write down the values of a and b . 2

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(c) Is it safe to administer this anti-biotic over an extended period of time? 4

Functions and Graphs

1. (a) Functions f and g are defined on suitable domains by

$$f(x) = 2x^2 + 5 \text{ and } g(x) = x - 1$$

Find $f(g(x))$.

2

- (b) Sketch the curve with equation $y = f(g(x))$.

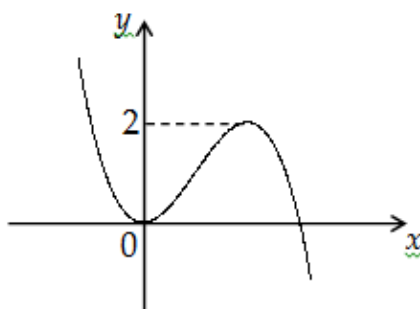
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2. Given that $h(x) = \frac{1}{x^2 - 16}$, what is the largest possible domain for h ?

2

3. The diagram shows the graph of $y = f(x)$.

2



Draw the diagram of the graph of $y = 2 - f(x)$?

4. Two functions f and g are defined on the set of real numbers by

$$f(x) = 2x + k \text{ and } g(x) = x^2 - 2k, \text{ where } k \neq 0$$



- (a) Find (i) $f(g(x))$;

(ii) $g(f(x))$.

3

- (b) Find the value of k for which $f(g(x)) = g(f(x))$ has equal roots.

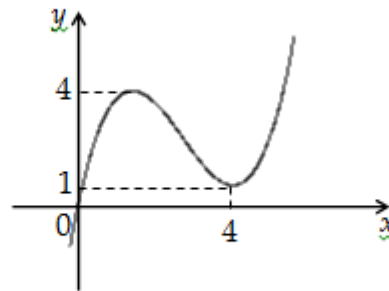
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5. Functions f and g are defined on the set of real numbers by 2

$$f(x) = x^2 + 1 \text{ and } g(x) = 3x - 5$$

What is the value of $g(f(-1))$?

6. The diagram shows the part of the graph of the cubic $y = f(x)$. 2



Draw the graph of $y = 4 - f(x)$?

Differentiation

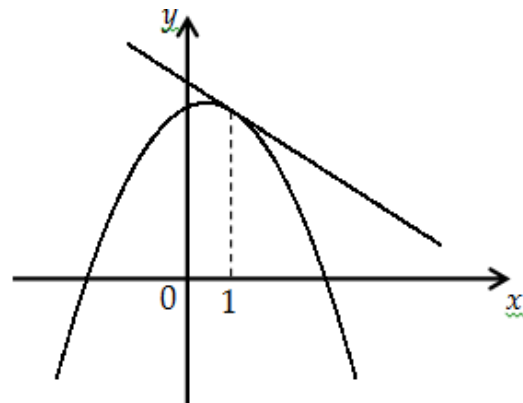
1. What is the derivative of $\frac{x^3 - 2}{3x}$ with respect to x ? 2

2. Find the stationary points on the curve given by $y = x^3 - 9x^2 + 24x - 2$ and determine their nature. 7

3. Find the equation of the tangent to the parabola with equation

$$y = 6 + x - x^2$$

at the point $(1, 6)$.



4. Given that $y = 4 \sin(3x - 2)$, find $\frac{dy}{dx}$. 4
2

5. Find the values of x for which the function $f(x) = 5 + 24x + 3x^2 - x^3$ is decreasing. 5

6. A function f is defined by $f(x) = 2x^3 - 3x^2$, where x is a real number.
(a) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x and y -axes. 3

(b) Find the stationary points on the curve $y = f(x)$ and determine their nature. 6

(c) Sketch the curve $y = f(x)$. 2

7. Given that $f(x) = \frac{1}{3x^5}$, find $f'(x)$. 2

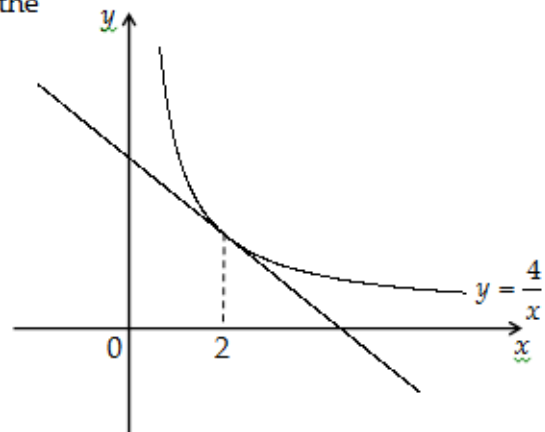
8. Given that $f(x) = 4 \sin 3x$, find $f'\left(\frac{\pi}{6}\right)$. 2

9. Find the equation of the tangent to the curve with equation



$$y = \frac{4}{x}$$

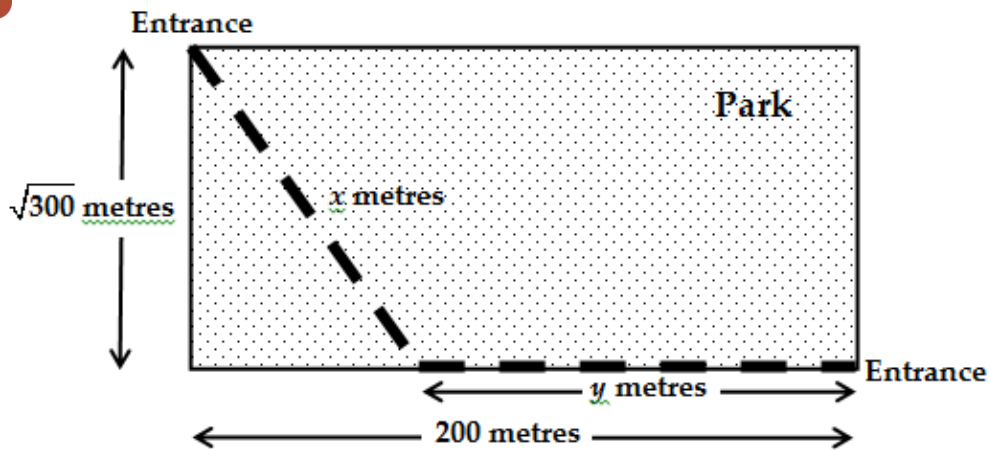
at the point where $x = 2$.



6

Differentiation (Optimisation)

1. A path connecting the two entrances, at opposite corners of the park, is to be laid through the park as shown.



The cost per metre of laying the path through the park is twice the cost, per metre, of laying the path along the perimeter.

- (a) Show that the total cost of laying this path can be modelled by

$$C(x) = 2x + 200 - \sqrt{x^2 - 300} \quad 3$$

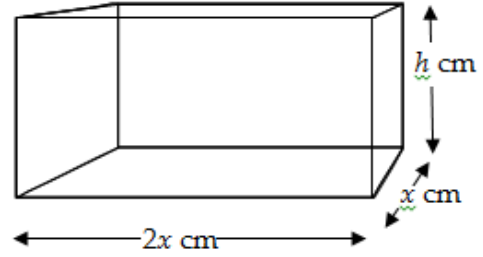
- (b) Find the value of x which would minimise the cost of laying the path. 6



2. A closed wooden box, in the shape of a cuboid, is constructed from a sheet of wood of area 600 cm^2 .

The base of the box measures $2x \text{ cm}$ by $x \text{ cm}$.

The height of the box is $h \text{ cm}$.



- (a) Assuming the thickness of the sides of the box are negligible, show that the volume (in cubic centimetres) of the box is given by

$$V(x) = 200x - \frac{4}{3}x^3 \quad 3$$

- (b) (i) Calculate the value of x for which this volume is a maximum.

- (ii) Find the maximum volume of the box. 7



3. A manufacturer of executive desks estimates that the weekly cost, in £, of making x desks is given by $C(x) = x^3 - 6x^2 + 560x + 800$.

Each executive desk sells for £2000.



- (a) Show that the weekly profit made from making x desks is given by

$$P(x) = -x^3 + 6x^2 + 1440x - 800 \quad 2$$

- (b) (i) How many desks would the manufacturer have to make each week in order to maximise his profit?

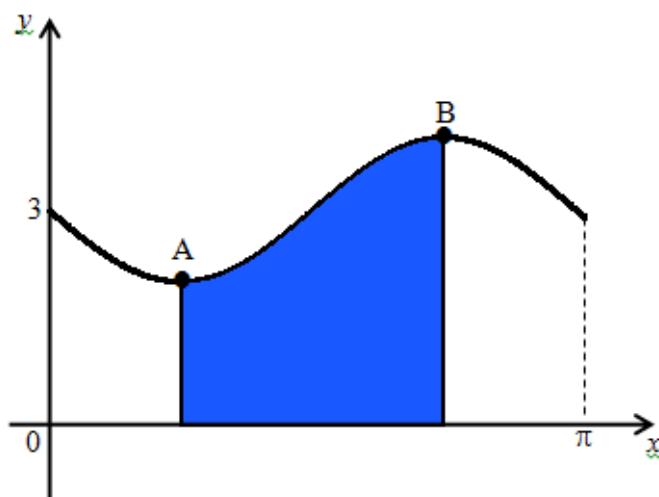
- (ii) What would his annual profit be? 8

Integration

1. Find $\int \frac{1}{2x^4} dx$. 2
2. Find $\int (3x-11)^5 dx$ 2
3. Find $\int (3x-11)^4 dx$. 2
4. Find $\int 5\cos(2x-1) dx$. 2
5. Find $\int (2x+5)^3 dx$. 2
6. Find $\int \left(x^{\frac{1}{2}} - x^{-2}\right) dx$ 2
7. The gradient of a tangent to a curve is given by $\frac{dy}{dx} = 2x - 3$. 4
If the curve passes through the point (4, 4), find its equation.

Integration (Area Under a Curve)

1. The graph of $y = 3 - \sin 2x$ has a minimum turning point at A and a maximum turning point at B.

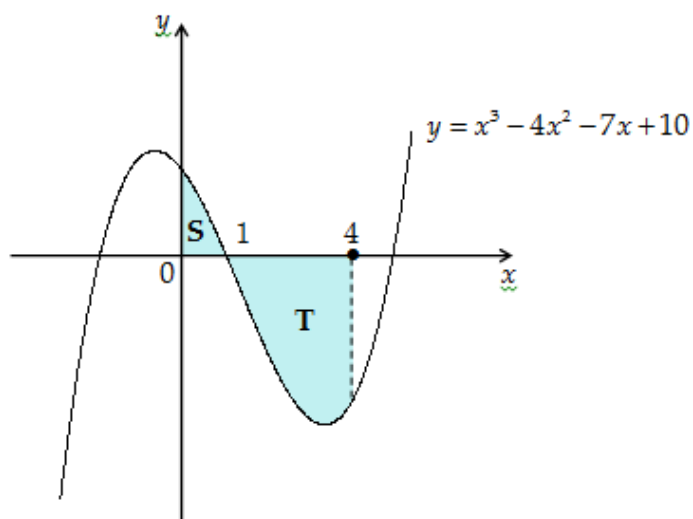


Calculate the shaded area in the diagram above.

7

2.

The graph shown has equation of the form $y = x^3 - 4x^2 - 7x + 10$.



Calculate the shaded area labelled S.

4

Hence find the total shaded area.

3

3. (a) Diagram 1 shows part of the graph with equation $y = x^3 - 5x^2 + 2x + 8$.

Calculate the shaded area.

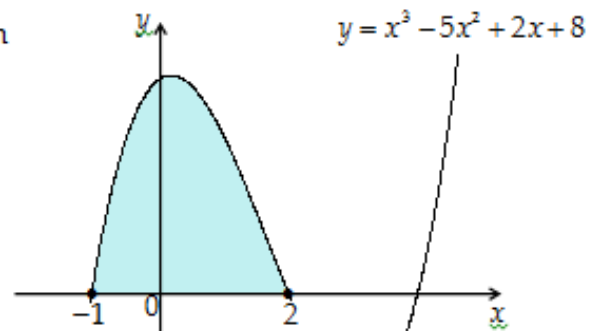


Diagram 1

5

- (b) Given that

$$\int_{-1}^p (x^3 - 5x^2 + 2x + 8) dx = 12 \cdot 13$$

find the total shaded area in diagram 2.

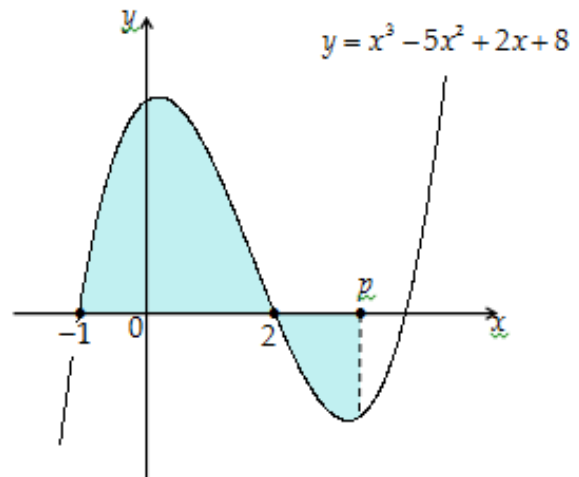


Diagram 2

2

Quadratic Functions

1. Find the discriminant of each equation and state the nature of the roots.
(a) $3x^2 - 7x + 2 = 0$ (b) $2x^2 - 3x + 4 = 0$ 4

2. For each equation find the value(s) for p so that the roots are equal.
(a) $x^2 - 8x + p = 0$ (b) $px^2 + 6x + 18 = 0$ 4

3. If $5 - 6x - x^2$ is written in the form $p - (x + q)^2$, what is the value of p ? 2

4. Express $2x^2 + 12x + 1$ in the form $a(x + b)^2 + c$. 3

Polynomials.

1. For the polynomial $6x^3 + 7x^2 + ax + b$,
 - $x + 1$ is a factor
 - 72 is the remainder when it is divided by $x - 2$.

(a) Determine the values of a and b . 4

(b) Hence factorise the polynomial completely. 3

2. Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$. Factorise this cubic fully. 4

3.
 - (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.
 - (ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.
 - (iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$. 6

4. (a) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.
(ii) Hence factorise $f(x)$ fully. 5

(b) Solve $2x^3 + x^2 - 8x + 5 = 0$. 1