NS Unit 3 Solutions

Graphs of Quadratics

Q1 $y = x^2 - 6x + 8$

$y = (x-2)(x-4)$

Roots: $x-2=0$ $\quad$ $x-4=0$

$x = 2$ $\quad$ $x = 4$

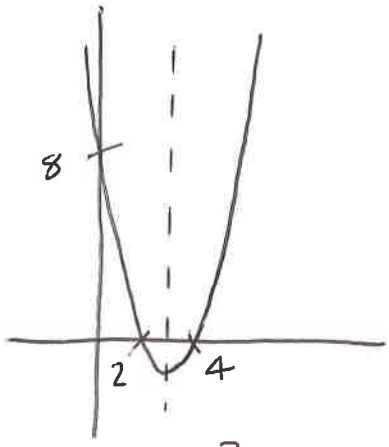
b) y-int 8

So, A = (0, 8)

B = (2, 0)

C = (4, 0)

c)



$x = 3$

Q2 $x^2 - 4x - 21$

$= (x+3)(x-7)$

$\times -21$

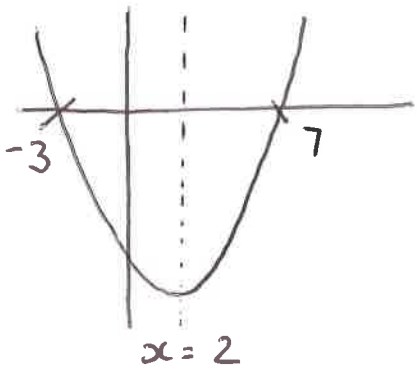
$+ -4$

b) $(x+3)(x-7) = 0$

$x + 3 = 0$ $\quad$ $x - 7 = 0$

$x = -3$ $\quad$ $x = 7$

c)



x coord of T.P is 2 i.e. $(2, ?)$ ↑ y

$y = x^2 - 4x - 21$

$= 2^2 - 4(2) - 21$

$= -25$

T.P has coord $(2, -25)$

ALTERNATIVE

Since coeff of $x^2 = 1$ i.e. $x^2 - 4x - 21$ we could "complete the square" to find T.P.

$x^2 - 4x - 21$

$= (x-2)^2 - 2^2 - 21$

$= (x-2)^2 - 4 - 21$

$= (x-2)^2 - 25$ $\quad \Rightarrow$ T.P. $(2, -25)$

Q3

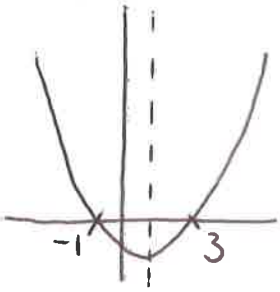
$y = x^2 - 2x - 3$

$y = (x-3)(x+1)$

$\times \; -3$
$+ \; -2$

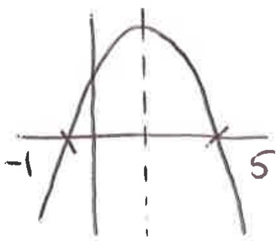
Roots: $x - 3 = 0$ → $x = 3$; $x + 1 = 0$ → $x = -1$

b)



$x = 1$

Q4



$x = 2$

b) x coord of T.P. is 2 ∴ $(2, ?)$ ← y

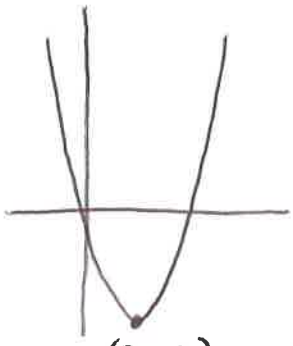
$y = 5 + 4x - x^2$

$= 5 + 4(2) - 2^2$

$= 9$

T.P. has coord $(2, 9)$

Q5



(2, -4) → $y = (x-2)^2 - 4$ So, $a = -2$ $b = -4$

Q6

$y = x^2 - 8x + 19$

$= (x-4)^2 - 4^2 + 19$

$= (x-4)^2 - 16 + 19$

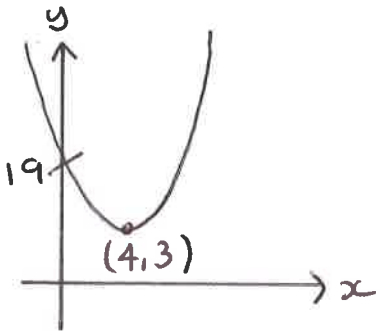
$= (x-4)^2 + 3$

b) $y = x^2 - 8x + 19$ Shape ∪ y-int 19

$y = (x-4)^2 + 3$ T.P. (4, 3)

Roots: $y = x^2 - 8x + 19$

CAN'T FACTORISE SINCE $b^2 - 4ac < 0$



Q7 $y = ax^2$

Subst (-3, 45)
(x, y)

$45 = a \times (-3)^2$

$45 = 9a$

(÷9) $a = 5$ i.e. $y = 5x^2$

Solving Quadratic Equations

Q1 a) $6y - y^2 = 0$

$y(6 - y) = 0$

$y = 0$ or $6 - y = 0$

$y = 6$

Sol^n is $y = 0$ and $y = 6$

b) $8x^2 - 12x = 0$

$4x(2x - 3) = 0$

$4x = 0$ or $2x - 3 = 0$

$x = 0$ $\quad x = \frac{3}{2}$

Sol^n is $x = 0$ and $x = \frac{3}{2}$

c) $4x^2 - 9 = 0$

$(2x + 3)(2x - 3) = 0$

$2x + 3 = 0$ or $2x - 3 = 0$

$x = -\frac{3}{2}$ $\quad x = \frac{3}{2}$

Sol^n is $x = -\frac{3}{2}$ and $x = \frac{3}{2}$

d) $x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$

$x - 4 = 0$ or $x - 2 = 0$

$x = 4$ $\quad x = 2$

Sol^n is $x = 2$ and $x = 4$

e) $x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ $\quad x = -2$

Sol^n is $x = -2$ and $x = 4$

f) $x^2 - 7x = 0$

$x(x - 7) = 0$

$x = 0$ or $x - 7 = 0$

$x = 7$

Sol^n is $x = 0$ and $x = 7$

Q2 $3x^2 + 5x - 7 = 0$

$a = 3$
$b = 5$
$c = -7$

$b^2 - 4ac = 5^2 - 4(3)(-7)$
$= 25 + 84$
$= 109$

Roots: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{5 \pm \sqrt{109}}{6}$

$x = \dfrac{5 - \sqrt{109}}{6}$ and $x = \dfrac{5 + \sqrt{109}}{6}$

$= -0.906\ldots$ $= 2.573\ldots$

$= -0.9$ $= 2.6$

Q3 $2x^2 + 3x - 7 = 0$

$a = 2$
$b = 3$
$c = -7$

$b^2 - 4ac = 3^2 - 4(2)(-7)$
$= 9 + 56$
$= 65$

Roots: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-3 \pm \sqrt{65}}{4}$

$x = \dfrac{-3 - \sqrt{65}}{4}$ and $x = \dfrac{-3 + \sqrt{65}}{4}$

$= -2.765\ldots$ $= 1.265\ldots$

$= -2.8$ $= 1.3$

Q4 a) $x^2 + 4x + 1$

$a = 1$
$b = 4$
$c = 1$

$b^2 - 4ac = 4^2 - 4(1)(1)$
$= 16 - 4$
$= 12$

Since $b^2 - 4ac > 0$ the roots are real and unequal.

b) $x^2 + x + 4$

$a = 1$
$b = 1$
$c = 4$

$b^2 - 4ac = 1^2 - 4(1)(4)$
$= 1 - 16$
$= -15$

Since $b^2 - 4ac < 0$ the roots are not real.

c) $x^2 + 3x - 2$

$a = 1$
$b = 3$
$c = -2$

$b^2 - 4ac = 3^2 - 4(1)(-2)$
$= 9 + 8$
$= 17$

Since $b^2 - 4ac > 0$ the roots are real and unequal

d) $3x^2 + 7x + 4$

$a =$
$b = 7$
$c = 4$

$b^2 - 4ac = 7^2 - 4(3)(4)$
$= 49 - 48$
$= 1$

Since $b^2 - 4ac > 0$ the roots are real and unequal

e) $b^2 - 4ac$
$= 3^2 - 4(2)(-2)$
$= 25$

Since $b^2 - 4ac > 0$ the roots are real/unequal

f) $b^2 - 4ac$
$= 9^2 - 4(4)(6)$
$= -15$

Since $b^2 - 4ac < 0$ the roots are not real.

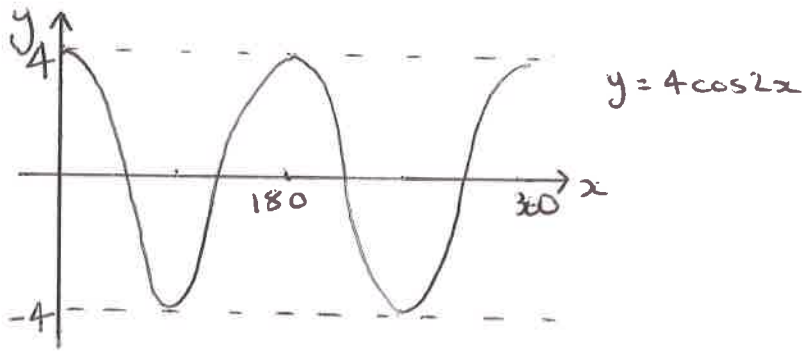
Trigonometric Graphs

Q1 $y = 4\sin 3x$ i.e. $a = 4$ $b = 3$

Q2 $y = 5\sin 3x$ i.e. $a = 5$ $b = 3$

Q3 $y = 5\cos 4x$ i.e. $a = 5$ $b = 4$

Q4



Q5 $y = \sin 2x$ Period: $360 \div 2 = 180°$

Solving Trig Eq^{n}

Q1 $4\sin x - 1 = 0$

$4\sin x = 1$

$\sin x = \frac{1}{4}$

$\sin x$ is positive in 

Q1 angle is $x = \sin^{-1}\left(\frac{1}{4}\right) = 14.477\ldots = 14.48$

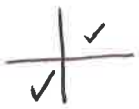
Q2 angle is $x = 180 - 14.48 = 165.52$

Sol^{n} is $x = 14.48^{\circ}$ and 165.52°

Q2 $2\tan x - 3 = 5$

$2\tan x = 8$

$\tan x = 4$

tan is positive in 

Q1 angle is $x = \tan^{-1}(4) = 75.963\ldots = 75.96$

Q3 angle is $x = 180 + 75.96 = 255.96$

Sol^{n} is $x = 75.96^{\circ}$ and 255.96°

Q3 $7\sin x - 3 = 0$

$7\sin x = 3$

$\sin x = \frac{3}{7}$

$\sin x$ is positive in 

Q1 angle is $x = \sin^{-1}\left(\frac{3}{7}\right) = 25.376\ldots = 25.38$

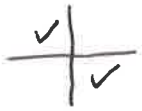
Q2 angle is $x = 180 - 25.38 = 154.62$

Sol^{n} is $x = 25.38^{\circ}$ and 154.62°

Q4 $4\tan x + 5 = 0$

$4\tan x = -5$

$\tan x = -\frac{5}{4}$

tan is negative in 

Q1 angle is $x = \tan^{-1}(\frac{5}{4}) = 51.34$

Q2 angle is $x = 180 - 51.34 = 128.66$

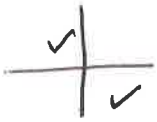
Q4 angle is $x = 360 - 51.34 = 308.66$

Soln is $x = 128.66°$ and $308.66°$

Q5 $2\tan x + 7 = 0$

$2\tan x = -7$

$\tan x = -\frac{7}{2}$

tan is negative in 

Q1 angle is $x = \tan^{-1}(\frac{7}{2}) = 74.054\ldots = 74.05$

Q2 angle is $x = 180 - 74.05 = 105.95$

Q4 angle is $x = 360 - 74.05 = 285.95$

Soln is $x = 105.95°$ and $285.95°$