Higher Maths Prelim Revision Booklet – Answers

The Straight Line

1. (a) $3x-y-2=0$ (b) $x+y-2=0$ (c) $(1,1)$
2. (a) $x=4$ (b) $3x-2y-9=0$ (c) $(4, 1\frac{1}{2})$
3. $3x-5y-7=0$
4. (a) $x-2y+4=0$ (b) $(-2,1)$
5. $\sqrt{3}$

Vectors

1. $k=-2$
2. (a) $\overrightarrow{KL}=\begin{pmatrix}2\\2\\4\end{pmatrix}$ $\overrightarrow{KM}=\begin{pmatrix}4\\-1\\-4\end{pmatrix}$ (b) $L\hat{K}M=110{\cdot}8^\circ$
3. $k=\frac{1}{5}$
4. (a) $\overrightarrow{AB}=\begin{pmatrix}2\\1\\1\end{pmatrix}$ $\overrightarrow{AC}=\begin{pmatrix}-3\\4\\2\end{pmatrix}$ (b) $B\hat{A}C=90^\circ$
5. (a) $B(8,5,0)$ $G(0,5,7)$ (b) $B\hat{E}G=72{\cdot}1^\circ$
6. $\overrightarrow{P_1P_2}=\begin{pmatrix}4\\2\\-12\end{pmatrix}$, $\overrightarrow{P_2N}=\begin{pmatrix}2\\1\\3\end{pmatrix}$ $\overrightarrow{P_1P_2}\neq k\overrightarrow{P_2N}$ $\Rightarrow P_1, P_2$ and N are not collinear
   $\Rightarrow$ Paxman will not hit Neptune.

The Circle

1. (a) $C_1(-3,-1)$, $r=1$ (b)(i) 10 (ii) 3
2. $(x+2)^2+(y-1)^2=58$
3. (a) $C(1,1)$, $r=\sqrt{22}$ (b)(i) show that $x=5, y=-3$ satisfies the equation
   (ii) $x+y-2=0$
   (c) $x-y-6=0$
4. (a) $C(1,3)$, $r=5$ (b) $(x-11)^2+(y-3)^2=100$
5. $(-3,4)$ is the only point of contact so the line is a tangent to the circle
6. $(x-6)^2+(y-10)^2=100$

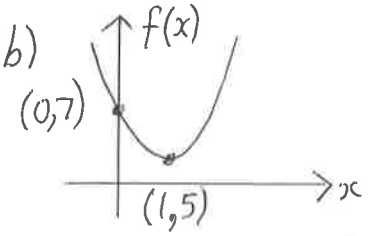
Recurrence Relations.

1. (a) 6, 10·4, 10·16 (b) $-1<0{\cdot}4<1$ (c) 0·6 2. (a) 14 (b) $6/7$
3. (a) $\frac{1}{3}$ (b)(i) $-1<\frac{1}{3}<1$ (ii) -9 4. (a) 50mg (b) $a=0{\cdot}2$, $b=250$
   (c) Yes, Limit = 312·5mg (which is <350mg)

Functions and Graphs

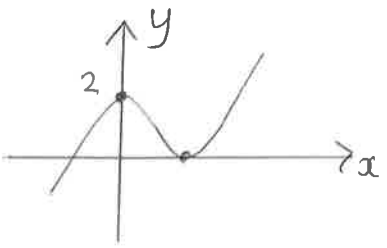
1. (a) $2x^2-4x+7$

(b)



2. $\{x : x \neq \pm 4, x \in R\}$

3.



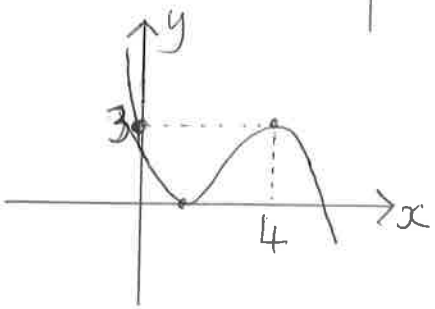
4. (a) (i) $2x^2-3k$

(ii) $4x^2+4xk+k^2-2k$

(b) $k=1$ (since $k \neq 0$)

5. 1

6.



Differentiation

1. $\frac{2}{3}x + \frac{2}{3x^2}$

2. (2,18) max. t.p.
(4,14) min. t.p.

3. $x+y-7=0$

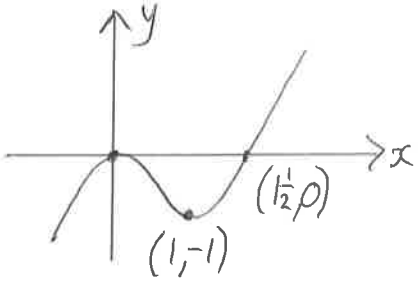
4. $12\cos(3x-2)$

5. $x<-2$ and $x>4$

6. (a) $(0,0)$, $(1\frac{1}{2},0)$

(b) $(0,0)$ max t.p.
$(1,-1)$ min t.p.

(c)



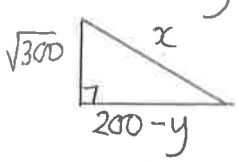
7. $\frac{-5}{3x^4}$

8. 0

9. $x+y-4=0$

Differentiation (Optimisation)

1. (a) proof. (Hint: $C = 2x+y$



$200-y=\sqrt{x^2-300}$

$\Rightarrow y = 200-\sqrt{x^2-300}$)

(b) 20

2. (a) proof (Hint: $V = 2x^3h$ where $600 = 4x^2+6xh$) (b) $\frac{400\sqrt{50}}{3} \simeq 942.8\ cm^3$

3. (a) proof (Hint: Profit = Value of sales − cost where cost = $2000x$)

(b) (i) 24 (ii) £23392

Integration

1. $-\frac{1}{6x^3}+c$ 2. $\frac{(3x-11)^6}{18}+c$ 3. $\frac{(3x-11)^5}{15}+c$

4. $10\sin(2x-1)+c$ 5. $\frac{(2x+5)^4}{8}+c$ 6. $\frac{4}{5}x^{5/4}+\frac{1}{x}+c$

7. $y=x^2-3x$

Integration (Area Under a Curve)

1. $\frac{3\pi}{2}$ units^2 2.(a) $5\frac{5}{12}$ units^2 (b) $48\frac{1}{6}$ units^2

3.(a) 17.75 units^2 (b) 23.37 units^2

Quadratic Functions

1(a) 25; two distinct rational roots (b) -23; no real roots.

2(a) 16 (b) $\frac{1}{2}$

3. $14-(x+3)^2$ so $p=14$

4. $2(x+3)^2-17$.

Polynomials

1.(a) $a=-1$, $b=-2$ (b) $(x+1)(3x+2)(2x-1)$

2. $(x-1)(x^2+4x+5)$ [note: x^2+4x+5 does not factorise; $b^2-4ac=-4$]

3. (i) $f(4)=0 \Rightarrow (x-4)$ is a factor

(ii) $(x-4)(x-2)(x+1)$

(iii) $x=4, 2$ or -1

4. (a)(i) $f(1)=0 \Rightarrow (x-1)$ is a factor

(ii) $(x-1)^2(2x+5)$

(b) $x=1$ or $-\frac{5}{2}$