Recurrence Relations

Q1 $\quad u_{n+1} = au_n + b \qquad u_0 = 10 \quad u_1 = 30 \quad u_2 = 46$

$u_1 = au_0 + b \Rightarrow 30 = 10a + b \quad (1)$

$u_2 = au_1 + b \Rightarrow 46 = 30a + b \quad (2)$

$(2) - (1) \qquad 16 = 20a$

$(\div 20) \qquad a = 0 \cdot 8$

Subs $a = 0 \cdot 8$ in (1) $\qquad 10(0 \cdot 8) + b = 30$

$8 + b = 30$

$b = 22$

So, $\quad u_{n+1} = 0 \cdot 8 u_n + 22$

$u_3 = 0 \cdot 8 u_2 + 22$

$= 0 \cdot 8(46) + 22$

$= 58 \cdot 8$

$u_4 = 69.04$

$u_5 = 77.232$

Q2 $u_{n+1} = au_n + b$ $\quad u_0 = 2000 \quad u_1 = 2500 \quad u_2 = 2980$

$u_1 = au_0 + b \Rightarrow 2500 = 2000a + b \quad (1)$

$u_2 = au_1 + b \Rightarrow 2980 = 2500a + b \quad (2)$

$(2) - (1) \quad 480 = 500a$

$(\div 500) \quad a = 0{\cdot}96$

Subst $a = 0{\cdot}96$ in (1)

$$2000(0{\cdot}96) + b = 2500$$
$$1920 + b = 2500$$
$$b = 580$$

So, $a = \underline{\underline{0{\cdot}96}}$ $\quad b = \underline{\underline{580}}$ $\quad \underline{\underline{u_{n+1} = 0{\cdot}96u_n + 580}}$

b) $L = \dfrac{b}{1-a}$ $\quad$ A limit exists since $-1 < 0{\cdot}96 < 1$

$= \dfrac{580}{1 - 0{\cdot}96}$

$= 14500$

Since $\underline{\underline{14500 > 13000}}$ conservation measures will end.

Q3 $\quad u_{n+1} = -\frac{1}{2}u_n \qquad u_0 = -16.$

$$u_1 = -\frac{1}{2}u_0 = -\frac{1}{2}(-16) = \underline{\underline{8}}$$

$$u_2 = -\frac{1}{2}u_1 = -\frac{1}{2}(8) = \underline{\underline{-4}}$$

b) $\quad V_{n+1} = pV_n + q \qquad V_1 = 4 \quad V_2 = 5 \quad V_3 = 7$

$V_2 = pV_1 + q$

$5 = 4p + q$ ①

$V_3 = pV_2 + q$

$7 = 5p + q$ ②

② − ① $\quad 2 = p$

Subst $p = 2$ in ①

$4p + q = 5$

$8 + q = 5$

(−8) $\quad q = -3$

So, $\underline{\underline{p = 2}} \quad \underline{\underline{q = -3}}$ i.e. $\underline{\underline{V_{n+1} = 2V_n - 3}}$

c) i) limit exists for (a) i.e. $u_{n+1} = -0.5u_n$

since $-1 < -0.5 < 1$

$$l = \frac{b}{1-a} = \frac{0}{1+0.5} = \underline{\underline{0}}$$

ii) since $2 > 1$ a limit does not exist for (b).