

Straight Line

Q1/ $3y + 2x = 6$

$(-2x)$ $3y = 6 - 2x$

$(\div 3)$ $y = 2 - \frac{2}{3}x$

The gradient of any line parallel to this will be $-\frac{2}{3}$.

Q2/ $2x - 3y - 6 = 0$

$(+3y)$ $2x - 6 = 3y$

$3y = 2x - 6$

$(\div 3)$ $y = \frac{2}{3}x - 2 \Rightarrow m = \frac{2}{3}$

L is perpendicular, so has gradient $-\frac{3}{2}$
since $m \times m_{\perp} = -1$.

Q3/ $x + 2y = 1$

$(-x)$ $2y = 1 - x$

$(\div 2)$ $y = \frac{1}{2} - \frac{1}{2}x \Rightarrow m = -\frac{1}{2}$

So, l_1 has gradient 2 since $m \times m_{\perp} = -1$.

$m = 2$

$a \quad b$
 $(1, 10)$

$y - b = m(x - a)$

$y - 10 = 2(x - 1)$

$y - 10 = 2x - 2$

$(+10)$ $y = \underline{\underline{2x + 8}}$

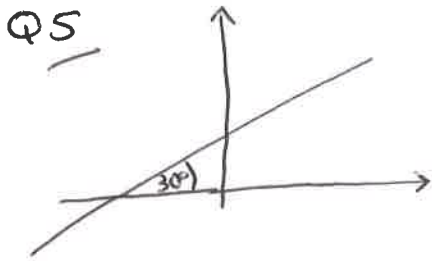
or $\underline{\underline{2x - y + 8 = 0}}$

Q4

$$3x + y + 1 = 0$$

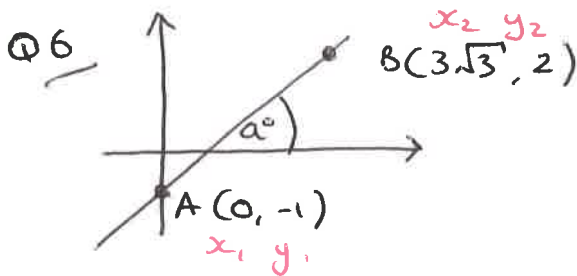
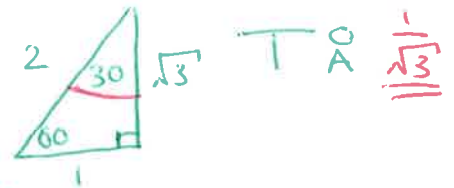
$$\begin{aligned} (-3x) \quad y + 1 &= -3x \\ (-1) \quad y &= -3x - 1 \Rightarrow m = -3 \end{aligned}$$

A line perpendicular to the above will have a gradient of $m = \frac{1}{3}$ so $a = \frac{1}{3}$ in the eqⁿ, $y = ax + 4$.



$$\begin{aligned} m &= \tan \theta \\ &= \tan 30^\circ \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

It is expected that you can do this without the use of a calculator using the Exact Value Triangle.



$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-1)}{3\sqrt{3} - 0} \\ &= \frac{3}{3\sqrt{3}} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} m &= \tan \theta \\ \tan \theta &= \frac{1}{\sqrt{3}} \\ \theta &= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \\ &= 30^\circ \end{aligned}$$

Q7

$$y = 2x + 4$$

gradient = 2

$$\begin{aligned} m &= \tan \theta \\ \tan \theta &= 2 \\ \theta &= \tan^{-1}(2) \\ &= 63.4^\circ \end{aligned}$$

$$\begin{aligned} x + y &= 13 \\ y &= 13 - x \end{aligned}$$

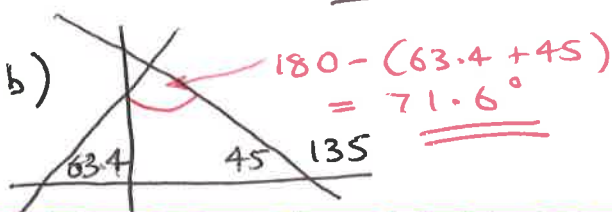
gradient = -1

$$\begin{aligned} m &= \tan \theta \\ \tan \theta &= -1 \\ \theta &= \tan^{-1}(1) \\ &= 45^\circ \end{aligned}$$

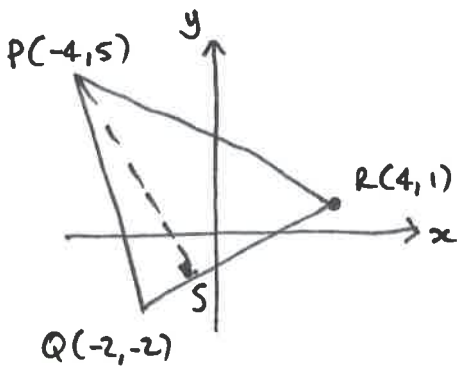
When the value is negative this means the answer will be larger than 90°

So, obtuse angle is $180 - 45^\circ = 135^\circ$

don't use negative sign.



Q8



GRADIENT

$$m_{QR} = \frac{1 - (-2)}{4 - (-2)}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

$\Rightarrow m_{PS} = -2$ since $m \times m_{\perp} = -1$.

POINT $(-4, 5)$

EQUATION

$$y - b = m(x - a)$$

$$y - 5 = -2(x - (-4))$$

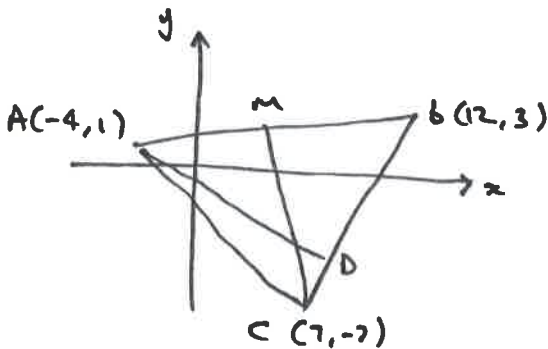
$$y - 5 = -2(x + 4)$$

$$y - 5 = -2x - 8$$

$$y = -2x - 3$$

(+5)

Q9



(a) Midpoint M = $\left(\frac{-4+12}{2}, \frac{1+3}{2}\right)$

$$= \left(\frac{8}{2}, \frac{4}{2}\right)$$

$$= \underline{\underline{(4, 2)}}$$

Gradient

x_1, y_1
C (7, -7)
 x_2, y_2
M (4, 2)

$$m_{CM} = \frac{2 - (-7)}{4 - 7}$$

$$= \frac{9}{-3}$$

$$= -3$$

Equation

$(4, 2)$

$m = -3$

$$y - b = m(x - a)$$

$$y - 2 = -3(x - 4)$$

$$y - 2 = -3x + 12$$

$$(+3x) \quad 3x + y - 2 = 12$$

$$(+2) \quad \underline{\underline{3x + y = 14}}$$

↑
left in this form as I will be solving simultaneously in part (c)

b) Gradient CB

$$\begin{matrix} x_1, y_1 \\ (12, 3) \end{matrix}$$

$$\begin{matrix} x_2, y_2 \\ (7, -7) \end{matrix}$$

$$m = \frac{-7 - 3}{7 - 12}$$

$$= \frac{-10}{-5}$$

$$= 2 \Rightarrow$$

$$m_{AD} = \underline{\underline{-\frac{1}{2}}} \text{ since } m \times m_{\perp} = -1$$

Point $(-4, 1)$

Equation

$$y - b = m(x - a)$$

$$y - 1 = -\frac{1}{2}(x - (-4))$$

$$y - 1 = -\frac{1}{2}(x + 4)$$

$$(\times 2) \quad 2y - 2 = -(x + 4)$$

$$2y - 2 = -x - 4$$

$$(\text{+}x) \quad x + 2y - 2 = -4$$

$$(\text{+}2) \quad \underline{\underline{x + 2y = -2}}$$

$$c) \quad 3x + y = 14 \quad \textcircled{1}$$

$$x + 2y = -2 \quad \textcircled{2}$$

$$\textcircled{1} \times 2 \quad 6x + 2y = 28 \quad \textcircled{3}$$

$$\textcircled{2} \times 1 \quad x + 2y = -2 \quad \textcircled{4}$$

$$\textcircled{3} - \textcircled{4} \quad 5x = 30$$

$$(\div 5) \quad x = 6$$

Subst $x = 6$ in $\textcircled{2}$

$$x + 2y = -2$$

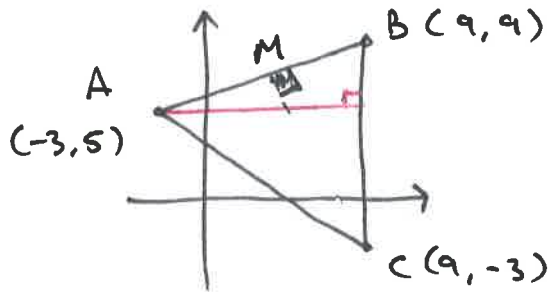
$$6 + 2y = -2$$

$$(\text{-}6) \quad 2y = -8$$

$$(\div 2) \quad y = -4$$

The point of intersection is $\underline{\underline{(6, -4)}}$

Q10



a) Gradient BC = $\frac{-3-9}{9-9} = \frac{-12}{0} = ?!$

x_1, y_1
(9, 9)

x_2, y_2
(9, -3)

HELP!!

Notice that B and C form a vertical line where x will always equal 9

There is no need to calculate the gradient here.

The eqⁿ is $x = 9$

b) The altitude from A has been added to the diagram above - do this on yours too!

GRADIENT since this is a horizontal line the gradient is 0.
I know it is horizontal since it meets a vertical line at a right angle. The eqⁿ is $y = 5$

c) Midpoint AB

A (-3, 5)

B (9, 9)

$$\left(\frac{-3+9}{2}, \frac{5+9}{2} \right)$$

$$= \left(\frac{6}{2}, \frac{14}{2} \right)$$

$$= (3, 7)$$

Please note that the perpendicular bisector of AB doesn't necessarily go through "C".

Gradient AB

A (-3, 5)

B (9, 9)

$$m = \frac{9-5}{9-(-3)}$$

$$= \frac{4}{12}$$

$$= \frac{1}{3} \Rightarrow m_{\perp} = -3 \text{ since } m \times m_{\perp} = -1$$

Equation

(3, 7)

$m = -3$

Eqⁿ

$$-b = m(x-a)$$

$$-7 = -3(x-3)$$

$$-7 = -3x + 9$$

$$y = \underline{\underline{-3x + 16}}$$

d) Perpendicular bisector : $y = -3x + 16$

Altitude : $y = 5$

To find intersection point $-3x + 16 = 5$

$$(-16) \quad -3x = -11$$

$$(\div -3) \quad x = \frac{11}{3}$$

i.e. (x, y) . Since we already know that $y = 5$

the point is $(\frac{11}{3}, 5)$

Vectors

Q1 $p = 3\underline{i} - 3\underline{j} + 2\underline{k}$

$$\begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix}$$

$q = 4\underline{i} - \underline{j} + \underline{k}$

$$\begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

$r = 4\underline{i} - 2\underline{j} + 3\underline{k}$

$$\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$$

a) $2p - q + r$

$$= 2 \begin{pmatrix} 3 \\ -3 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -6 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -7 \\ 6 \end{pmatrix} \rightarrow \underline{\underline{6\underline{i} - 7\underline{j} + 6\underline{k}}}$$

b) $|2p - q + r|$

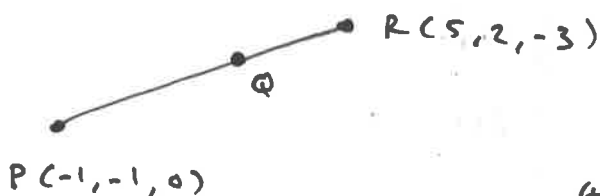
$$= \sqrt{6^2 + (-7)^2 + 6^2}$$

$$= \sqrt{36 + 49 + 36}$$

$$= \sqrt{121}$$

$$= \underline{\underline{11}}$$

Q2



$\circ \circ \circ \vec{PQ} = \frac{2}{3} \vec{PR}$

$$q - p = \frac{2}{3} r - \frac{2}{3} p$$

$$q = \frac{2}{3} r + \frac{1}{3} p$$

$$= \frac{2}{3} \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{10}{3} \\ \frac{4}{3} \\ -2 \end{pmatrix} + \begin{pmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{9}{3} \\ \frac{3}{3} \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

i.e. Q is $(\underline{\underline{3, 1, -2}})$

Q3 $P(2, 2, 3)$ $Q(4, 4, 1)$ $R(5, 5, 0)$

$$\vec{PQ} = \underline{q} - \underline{p}$$

$$= \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$$

$$= 2 \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\vec{QR} = \underline{r} - \underline{q}$$

$$= \begin{pmatrix} 5 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

Since $\vec{PQ} = 2\vec{QR}$ vectors parallel and since Q is a common point P, Q, R are collinear.

Ratio $\vec{PQ} : \vec{QR}$

$$\underline{2 : 1}$$

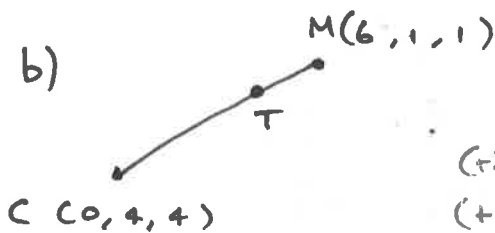
Q4 a) $A(4, -1, 3)$ $B(8, 3, -1)$ $C(0, 4, 4)$ $D(-4, 0, 8)$

$$M = \left(\frac{4+8}{2}, \frac{-1+3}{2}, \frac{3+(-1)}{2} \right)$$

$$= \left(\frac{12}{2}, \frac{2}{2}, \frac{2}{2} \right)$$

$$= \underline{\underline{(6, 1, 1)}}$$

b)



$$\vec{CT} = 2\vec{TM}$$

$$\begin{matrix} \underline{t} & - & \underline{c} & = & 2\underline{m} & - & 2\underline{t} \\ (+2t) & 3\underline{t} & - & \underline{c} & = & 2\underline{m} \end{matrix}$$

$$\begin{matrix} (+c) & 3\underline{t} & = & 2\underline{m} & + & \underline{c} \end{matrix}$$

$$3\underline{t} = 2 \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix}$$

$$3\underline{t} = \begin{pmatrix} 12 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \\ 4 \end{pmatrix}$$

$$3\underline{t} = \begin{pmatrix} 12 \\ 6 \\ 6 \end{pmatrix}$$

$$\therefore \underline{t} = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} \quad \text{i.e. } \underline{\underline{(4, 2, 2)}}$$

c) $B(8, 3, -1)$ $T(4, 2, 2)$ $D(-4, 0, 8)$

$$\begin{aligned} \vec{BT} &= \underline{t} - \underline{b} & \vec{TD} &= \underline{d} - \underline{t} \\ &= \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 8 \\ 3 \\ -1 \end{pmatrix} & &= \begin{pmatrix} -4 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} \\ &= \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix} & &= \begin{pmatrix} -8 \\ -2 \\ 6 \end{pmatrix} \\ & & &= 2 \vec{BT} \end{aligned}$$

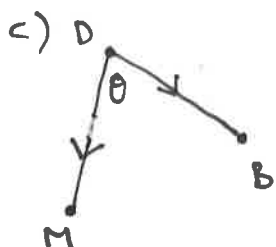
Since $\vec{TD} = 2\vec{BT}$ vectors parallel and since T is a common point B, T, D are collinear.

$$\begin{aligned} \vec{BT} : \vec{TD} \\ 1 : 2 \\ \underline{\underline{\hspace{1cm}}}\end{aligned}$$

QS $B(4, 4, 0)$

b) $D(2, 2, 6)$ $B(4, 4, 0)$ $M(2, 0, 0)$

$$\begin{aligned} \vec{DB} &= \underline{b} - \underline{d} & \vec{DM} &= \underline{m} - \underline{d} \\ &= \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} & &= \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix} \\ &= \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix} & &= \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix} \\ & \underline{\underline{\hspace{1cm}}} & & \underline{\underline{\hspace{1cm}}}\end{aligned}$$



$$\begin{aligned} \cos \theta &= \frac{\vec{DB} \cdot \vec{DM}}{|\vec{DB}| |\vec{DM}|} \\ &= \frac{32}{\sqrt{44} \sqrt{40}} \\ \theta &= \cos^{-1} \left(\frac{32}{\sqrt{44} \sqrt{40}} \right) \\ &= \underline{\underline{40.3^\circ}} \quad (\text{1dp}) \end{aligned}$$

$$\begin{aligned} |\vec{DB}| &= \sqrt{2^2 + 2^2 + (-6)^2} \\ &= \sqrt{44} \\ |\vec{DM}| &= \sqrt{0^2 + (-2)^2 + (-6)^2} \\ &= \sqrt{40} \\ \vec{DB} \cdot \vec{DM} &= 2(0) + 2(-2) + (-6)(-6) \\ &= 32 \end{aligned}$$

Q6

(a) M $(0, 1, 0)$

N $(4, 2, 2)$

(b) \vec{VM}

\vec{VN}

$= \underline{M} - \underline{V}$

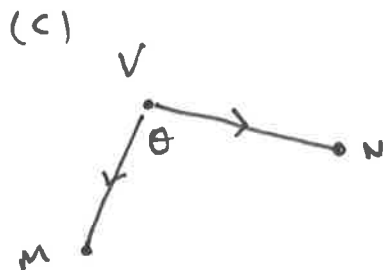
$= \underline{N} - \underline{V}$

$= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}}}$

$= \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$



$\cos \theta = \frac{\vec{VM} \cdot \vec{VN}}{|\vec{VM}| |\vec{VN}|}$

$= \frac{3}{\sqrt{10} \sqrt{17}}$

$\theta = \cos^{-1} \left(\frac{3}{\sqrt{10} \sqrt{17}} \right)$

$= \underline{\underline{76.7^\circ}}$

•

$\vec{VM} \cdot \vec{VN} = 0(4) + (-1)(0) + (-3)(-1)$
 $= 3$

$|\vec{VM}| = \sqrt{0^2 + (-1)^2 + (-3)^2}$
 $= \sqrt{10}$

$|\vec{VN}| = \sqrt{4^2 + 0^2 + (-1)^2}$
 $= \sqrt{17}$

Q7 \vec{AB}

$$= \underline{b} - \underline{a}$$

$$= \begin{pmatrix} 3 \\ 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

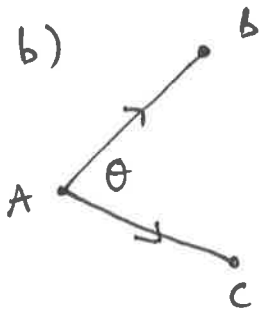
$$= \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$$

\vec{AC}

$$= \underline{c} - \underline{a}$$

$$= \begin{pmatrix} 6 \\ 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$$



$$\cos \theta = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}| |\vec{AC}|}$$

$$= \frac{43}{\sqrt{54} \sqrt{90}}$$

$$\theta = \cos^{-1} \left(\frac{43}{\sqrt{54} \sqrt{90}} \right)$$

$$= \underline{\underline{51.9^\circ}} \text{ (1dp)}$$

$\vec{AB} \cdot \vec{AC}$

$$= 1(4) + 7(7) + 2(-5)$$

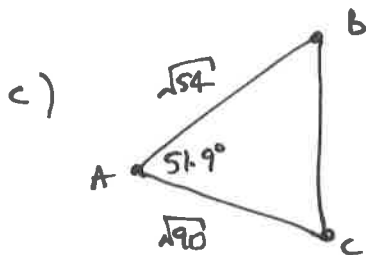
$$= 43$$

$$|\vec{AB}| = \sqrt{1^2 + 7^2 + 2^2}$$

$$= \sqrt{54}$$

$$|\vec{AC}| = \sqrt{4^2 + 7^2 + (-5)^2}$$

$$= \sqrt{90}$$



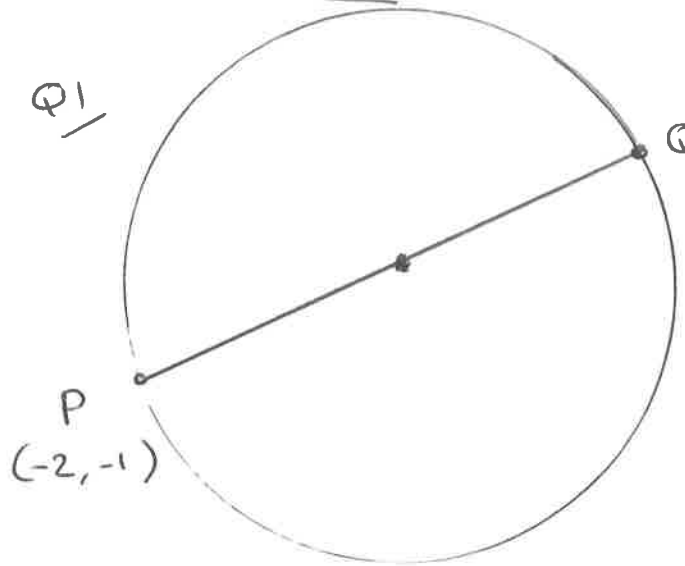
$$\text{Area} = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times \sqrt{54} \times \sqrt{90} \times \sin 51.9$$

$$= 27.430\dots$$

$$= \underline{\underline{27.4}}$$

The Circle



RADIUS

$$\begin{aligned}
 PQ &= \sqrt{(-2 - 4)^2 + (-1 - 5)^2} \\
 &= \sqrt{(-6)^2 + (-6)^2} \\
 &= \sqrt{36 + 36} \\
 &= \sqrt{64} \\
 &= 8
 \end{aligned}$$

Diameter = 8 units

Radius = 4 units

CENTRE

$$\begin{aligned}
 \text{midpoint } PQ &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{-2 + 4}{2}, \frac{-1 + 5}{2} \right) \\
 &= \left(\frac{2}{2}, \frac{4}{2} \right) \\
 &= \underline{\underline{(1, 2)}}
 \end{aligned}$$

EQUATION OF CIRCLE

radius = 4

centre = (1, 2)

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\underline{\underline{(x - 1)^2 + (y - 2)^2 = 16}}$$

Q2 $x^2 + y^2 - 10x - 4y + 12 = 0$

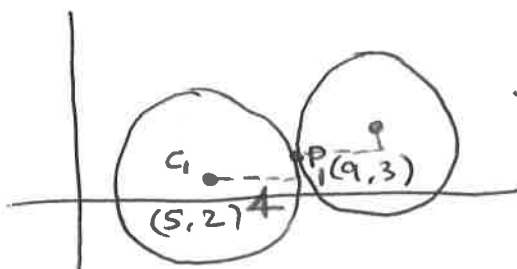
$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -10 \quad 2f = -4 \quad c = 12$$

$$g = -5 \quad f = -2$$

centre of circle 1 $(-g, -f)$
(5, 2)

$$\begin{aligned}
 \text{radius} &= \sqrt{g^2 + f^2 - c} \\
 &= \sqrt{5^2 + 2^2 - 12} \\
 &= \sqrt{25 + 4 - 12} \\
 &= \underline{\underline{\sqrt{17}}}
 \end{aligned}$$



→ Centre of circle 2 is (13, 4)
 Radius of circle 2 is $\sqrt{17}$

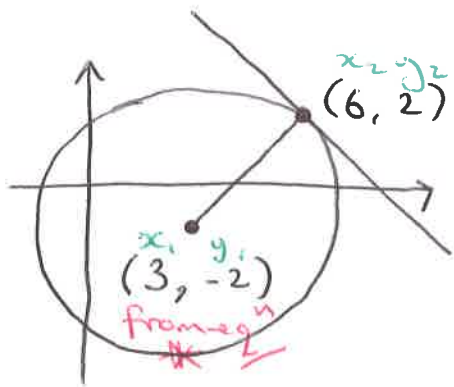
} Eqⁿ of circle 2
 $(x - a)^2 + (y - b)^2 = r^2$
 $(x - 13)^2 + (y - 4)^2 = 17$

Q3 $(x-3)^2 + (y+2)^2 = 25$

Eqⁿ of tangent = Eqⁿ of line

USE $y-b = m(x-a)$

NEED gradient (m)
point (a, b)



$$m = \frac{2 - (-2)}{6 - 3} = \frac{4}{3}$$

Since radius and tangent are perpendicular the gradient of the tangent is $-\frac{3}{4}$ (since $m \times m_{\perp} = -1$)

EQUATION OF TANGENT

$$m = -\frac{3}{4}$$

$$y - b = m(x - a)$$

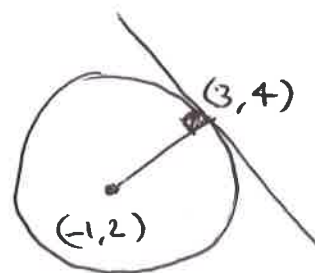
$$(6, 2) \quad y - 2 = -\frac{3}{4}(x - 6)$$

$$(x+1) \quad 4y - 8 = -3(x - 6)$$

$$4y - 8 = -3x + 18$$

$$(x+3) \quad 3x + 4y - 8 = 18$$

$$(-18) \quad \underline{\underline{3x + 4y - 26 = 0}}$$



Q4 $x^2 + y^2 + 2x - 4y - 15 = 0$
 $x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 2 \quad 2f = -4$ So centre is $(-g, -f) = (-1, 2)$
 $g = 1 \quad f = -2$

Gradient of radius = $\frac{4 - 2}{3 - (-1)} = \frac{2}{4} = \frac{1}{2} \Rightarrow$ Gradient of tangent = -2
 (since $m \times m_{\perp} = -1$)

EQUATION OF TANGENT

$$m = -2$$

$$y - b = m(x - a)$$

$$(3, 4) \quad y - 4 = -2(x - 3)$$

$$y - 4 = -2x + 6$$

$$(x+4) \quad \underline{\underline{y = -2x + 10}}$$

Q5 $x^2 + y^2 + 6x + 2y - 22 = 0$

Subst $x = 1$ and $y = 3$ i.e. $(1, 3)$ into above.

$$\begin{aligned} & x^2 + y^2 + 6x + 2y - 22 \\ &= 1^2 + 3^2 + 6(1) + 2(3) - 22 \\ &= 1 + 9 + 6 + 6 - 22 \\ &= 22 - 22 \\ &= 0 \end{aligned}$$

as required to show. Therefore $(1, 3)$ lies on the circle.

Q6 $2x - y + 6 = 0$

(+y) $y = 2x + 6$ Subst. this into circle eqⁿ:

$$x^2 + y^2 + 2x + 2y - 8 = 0$$

$$x^2 + (2x+6)^2 + 2x + 2(2x+6) - 8 = 0$$

$$x^2 + 4x^2 + 24x + 36 + 2x + 4x + 12 - 8 = 0$$

$$5x^2 + 30x + 40 = 0$$

$$(\div 5) \quad x^2 + 6x + 8 = 0$$

$$\begin{aligned} & \dots \text{C} \\ & (2x+6)^2 \\ &= (2x+6)(2x+6) \\ &= 4x^2 + 12x + 12x + 36 \\ &= 4x^2 + 24x + 36 \end{aligned}$$

Compare $ax^2 + bx + c = 0$

$$a = 1$$

$$b = 6$$

$$c = 8$$

$$b^2 - 4ac$$

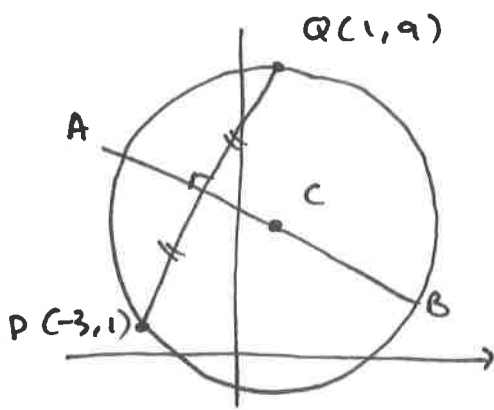
$$= 6^2 - 4(8)(1)$$

$$= 36 - 32$$

$$= 4$$

Since $b^2 - 4ac > 0$ there are two points of intersection.

Q7



$$\text{Gradient } PQ = \frac{1-9}{-3-1} = \frac{-8}{-4} = 2$$

$$\Rightarrow m_{AB} = -\frac{1}{2} \text{ since } m \times m_{\perp} = -1$$

$$\text{Midpoint } PQ = \left(\frac{1+(-3)}{2}, \frac{9+1}{2} \right)$$

$$= \left(\frac{-2}{2}, \frac{10}{2} \right)$$

$$= \underline{\underline{(-1, 5)}}$$

Equation
POINT $(-1, 5)$
GRADIENT

$$y - b = m(x - a)$$

$$y - 5 = -\frac{1}{2}(x - (-1))$$

$$y - 5 = -\frac{1}{2}(x + 1)$$

$$(*2) \quad 2y - 10 = -(x + 1)$$

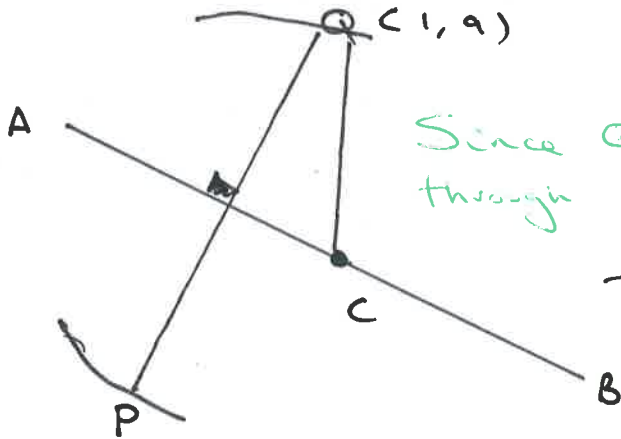
$$2y - 10 = -x - 1$$

$$x + 2y - 9 = 0$$

$$\left[\text{or } y = \frac{9}{2} - \frac{1}{2}x \right]$$

b) EQUATION OF A CIRCLE...

NEED: centre & radius.



Since QC is a vertical line going through $(1, 9)$ the eqⁿ of it is $x = 1$

This also means that the coordinate of C is $(1, ?)$

Since "C" lies on line AB with eqⁿ,

$$y = \frac{9}{2} - \frac{1}{2}x$$

$$y = \frac{9}{2} - \frac{1}{2}(1)$$

$$= \frac{8}{2}$$

$$= 4$$

i.e. C is $(1, 4)$

And since CQ is the radius, $r = 5$
(since C is $(1, 4)$ and Q is $(1, 9)$)

EQUATION
 $r = 5$
 $C = (1, 4)$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\underline{\underline{(x - 1)^2 + (y - 4)^2 = 25}}$$

(c) i) EQUATION OF TANGENT ^{at point} $(1, 9)$
gradient ?

Since $m = \text{"undefined"}$ and the radius and tangent meet at 90° (i.e. perpendicular) the gradient of the tangent at Q must be 0 (i.e. horizontal)

So the eqⁿ of the tangent at Q is $y = 9$

ii) Use eqⁿ of AB from part (a)

$$y = \frac{9}{2} - \frac{1}{2}x \quad \text{and} \quad y = 9$$

$$9 = \frac{9}{2} - \frac{1}{2}x$$

$$(42) \quad 18 = 9 - x$$

$$(-9) \quad 9 = -x$$

$$x = -9$$

i.e. coordinate of T is $(-9, 9)$

^o Before starting this question I would draw a diagram to picture a tangent kite.

