

## Recurrence Relations

1. For a sequence is defined by the recurrence relation  $u_{n+1} = au_n + b$  the terms  $u_0 = 10$ ,  $u_1 = 30$  and  $u_2 = 46$ . Use this information to determine the values of  $a$  and  $b$  and hence calculate  $u_5$ .

2. A wildlife reserve has introduced conservation measures to build up the population of an endangered mammal. Initially the reserve population of the mammal was 2000. By the end of the first year there were 2500 and by the end of the second year there were 2980.

It is believed that the population can be modelled by the recurrence relation:

$$u_{n+1} = au_n + b,$$

where  $a$  and  $b$  are constants and  $n$  is the number of years since the reserve was set up.

- (a) Use the information above to find the values of  $a$  and  $b$ .
- (b) Conservation measures will end if the population stabilises at over 13 000. Will this happen? Justify your answer.

3. (a) A sequence is defined by  $u_{n+1} = -\frac{1}{2}u_n$  with  $u_0 = -16$ .

Write down the values of  $u_1$  and  $u_2$ .

- (b) A second sequence is given by 4, 5, 7, 11, . . . .

It is generated by the recurrence relation  $v_{n+1} = pv_n + q$  with  $v_1 = 4$ .

Find the values of  $p$  and  $q$ .

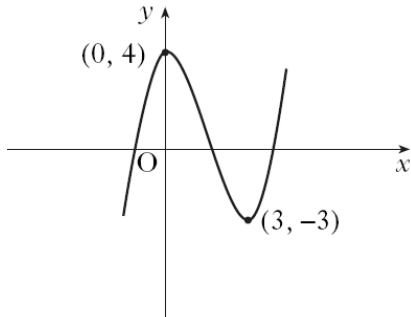
- (c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate this limit.

(ii) Why does the other sequence not have a limit?

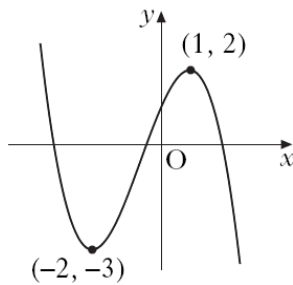
## Functions and Graphs

1. The diagram shows part of the graph of a function with equation  $y = f(x)$ .



Draw the graph of  $y = -f(x - 2)$ .

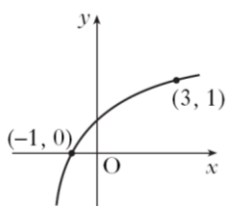
2. The diagram shows the graph of  $y = f(x)$ .



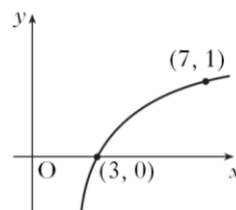
Which of the following shows the graph of  $y = f(x + 2) - 1$ ?

3. Which of the following graphs has equation  $y = \log_5(x - 2)$ ?

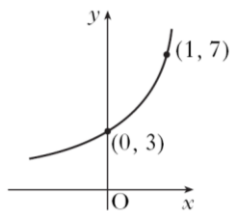
A



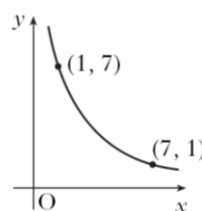
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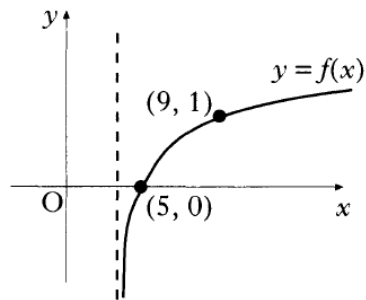
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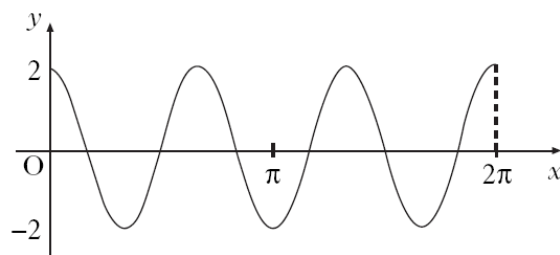
D



4. The function  $f$  is of the form  $f(x) = \log_b(x - a)$ .  
 The graph of  $y = f(x)$  is shown in the diagram.  
 (a) Write down the values of  $a$  and  $b$ .  
 (b) State the domain of  $f$ .

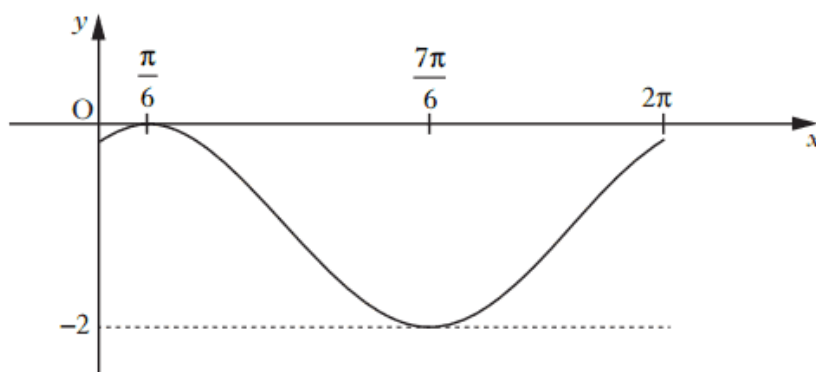


5. The diagram shows the graph with equation of the form  $y = a \cos bx$  for  $0 \leq x \leq 2\pi$ .



What is the equation of this graph?

- A  $y = 2 \cos 3x$   
 B  $y = 2 \cos 2x$   
 C  $y = 3 \cos 2x$   
 D  $y = 4 \cos 3x$
6. The diagram shows the curve with equation of the form  $y = \cos(x + a) + b$  for  $0 \leq x \leq 2\pi$ .



What is the equation of this curve?

- A  $y = \cos\left(x - \frac{\pi}{6}\right) - 1$   
 B  $y = \cos\left(x - \frac{\pi}{6}\right) + 1$   
 C  $y = \cos\left(x + \frac{\pi}{6}\right) - 1$   
 D  $y = \cos\left(x + \frac{\pi}{6}\right) + 1$

6. Functions  $f$  and  $g$ , defined on suitable domains, are given by  $f(x) = x^2 + 1$  and  $g(x) = 1 - 2x$ .

Find:

- (a)  $g(f(x))$ ;  
(b)  $g(g(x))$ .

7. A function  $f$  is given by  $f(x) = \sqrt{9 - x^2}$ .  
What is a suitable domain of  $f$ ?

- A  $x \geq 3$   
B  $x \leq 3$   
C  $-3 \leq x \leq 3$   
D  $-9 \leq x \leq 9$

8. Functions  $f(x) = \frac{1}{x-4}$  and  $g(x) = 2x + 3$  are defined on suitable domains.

- (a) Find an expression for  $h(x)$  where  $h(x) = f(g(x))$ .  
(b) Write down any restriction on the domain of  $h$ .

## Differentiation

1. If  $y = 3x^{-2} + 2x^{\frac{3}{2}}$ ,  $x > 0$ , determine  $\frac{dy}{dx}$ .

A  $-6x^{-3} + \frac{4}{5}x^{\frac{5}{2}}$   
B  $-3x^{-1} + 3x^{\frac{1}{2}}$   
C  $-6x^{-3} + 3x^{\frac{1}{2}}$   
D  $-3x^{-1} + \frac{4}{5}x^{\frac{5}{2}}$
2. Given that  $f(x) = \sqrt{x} + \frac{2}{x^2}$ , find  $f'(4)$ .
3. If  $s(t) = t^2 - 5t + 8$ , what is the rate of change of  $s$  with respect to  $t$  when  $t = 3$ ?

A  $-5$   
B  $1$   
C  $2$   
D  $9$
4. Given that  $y = 3\sin(x) + \cos(2x)$ , find  $\frac{dy}{dx}$ .
5. Given that  $f(x) = 4 \sin 3x$ , find  $f'(0)$ .

A  $0$   
B  $1$   
C  $12$   
D  $36$

6. If  $y = 3\cos^4 x$ , find  $\frac{dy}{dx}$ .

- A  $12\cos^3 x \sin x$
- B  $12\cos^3 x$
- C  $-12\cos^3 x \sin x$
- D  $-12\sin^3 x$

7. A curve has equation  $y = 5x^3 - 12x$ .

What is the gradient of the tangent at the point  $(1, -7)$ ?

- A  $-7$
- B  $-5$
- C  $3$
- D  $5$

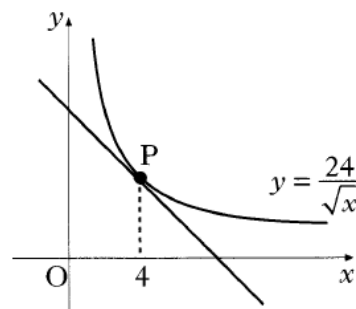
8. A tangent to the curve with equation  $y = x^3 - 2x$  is drawn at the point  $(2, 4)$

What is the gradient of this tangent?

- A  $2$
- B  $3$
- C  $4$
- D  $10$

9. The diagram shows the graph of  $y = \frac{24}{\sqrt{x}}$ ,  $x > 0$ .

Find the equation of the tangent at P, where  $x = 4$ .



- (a) Find the equation of the tangent to the curve with equation  $y = x^3 + 2x^2 - 3x + 2$  at the point where  $x = 1$ .
- (b) Show that this line is also a tangent to the circle with equation  $x^2 + y^2 - 12x - 10y + 44 = 0$  and state the coordinates of the point of contact.