Trigonometric Equations

1. What is the solution of the equation $2\sin x - \sqrt{3} = 0$ where $\frac{\pi}{2} \leq x \leq \pi$?

2. How many solutions does the equation

$$(4\sin x - \sqrt{5})(\sin x + 1) = 0$$

have in the interval $0 \leq x < 2\pi$?

3. Solve $2\cos x = \sqrt{3}$ for x, where $0 \leq x < 2\pi$.

Integration

1. Find $\int \frac{1}{3x^4}\,dx$, where $x \neq 0$.

2. Find $\int \left(4x^{\frac{1}{2}} + x^{-3}\right) dx$, where $x > 0$.

3. Find $\int \frac{4x^3 - 1}{x^2}\,dx$, $x \neq 0$.

4. Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$.

5. Find $\int_0^2 \sqrt{4x+1}\,dx$.

6. Find the value of $\int_0^2 \sin(4x+1)\,dx$.

7. Find $\int (2x^{-4} + \cos 5x)\,dx$.

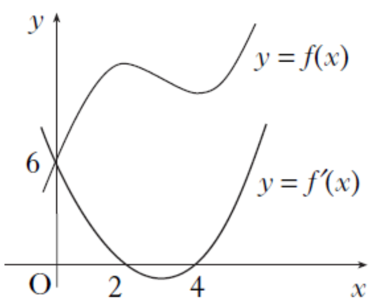
8. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x.

9. EXTENSION

The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).



(*a*) Given that $f'(x)$ is of the form $k(x-a)(x-b)$:

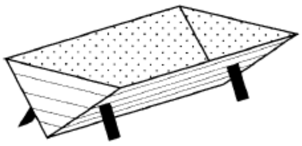
(i) write down the values of a and b;

(ii) find the value of k.

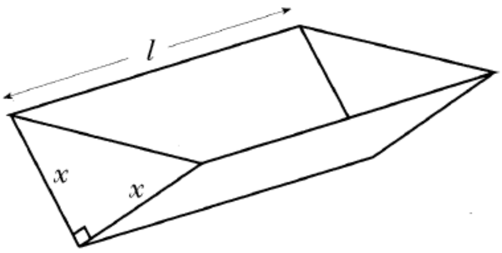
(*b*) Find the equation of the graph of the cubic function $y = f(x)$.

Optimisation and Area

1. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



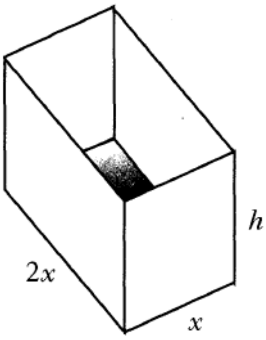
The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.



(a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$.

(b) Find the value of x which minimises this surface area.

2. An open cuboid measures internally x units by $2x$ units by h units and has an inner surface area of 12 units^2.

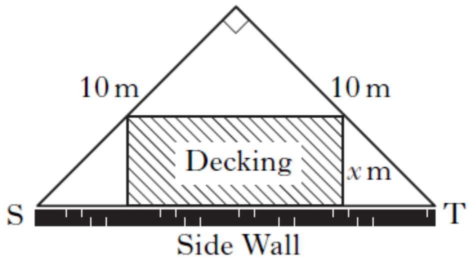


(a) Show that the volume, V units^3, of the cuboid is given by $V(x) = \frac{2}{3}x\left(6 - x^2\right)$.

(b) Find the exact value of x for which this volume is a maximum.

3. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2.$$

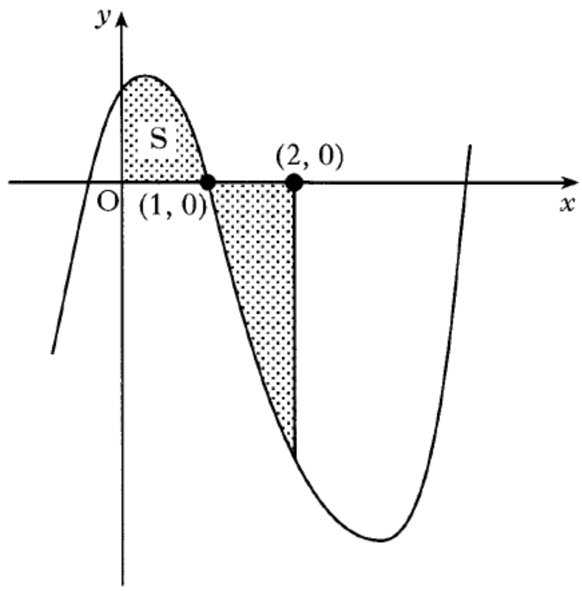
(*b*) Find the dimensions of the decking which maximises its area.

4. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

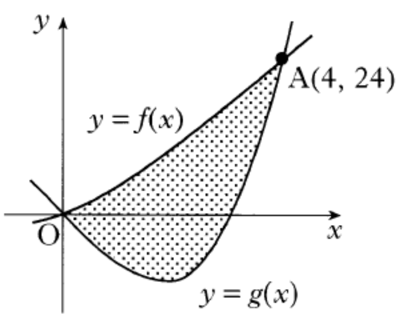
(*a*) Calculate the shaded area labelled S.

(*b*) Hence find the total shaded area.



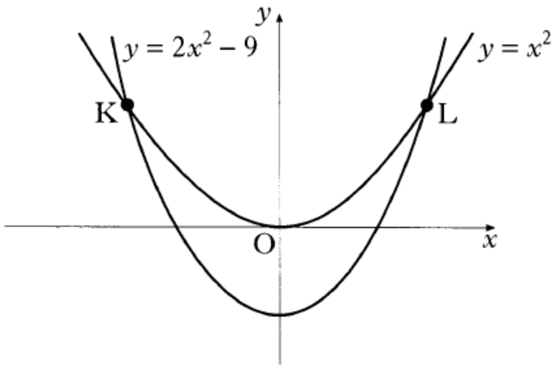
5. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

Find the shaded area enclosed between the curves.



6. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.

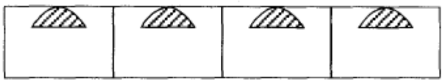


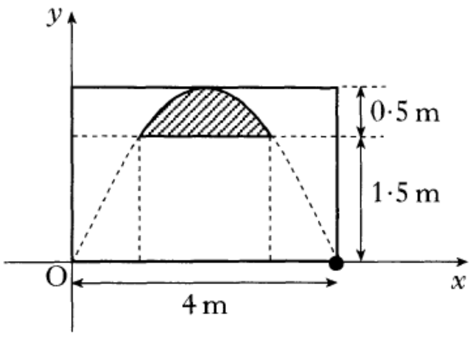
7. An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.

The second diagram shows one such window. The shaded part represents the glass.

The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$.

Find the area in square metres of the glass in one window.





8. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area.