Polynomials and Quadratics

1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.

(*a*) Show that $(x - 2)$ is a factor of $f(x)$.

(*b*) Express $f(x)$ in its fully factorised form. **4**

2. $f(x) = x^3 - x^2 - 5x - 3$.

(*a*) (i) Show that $(x + 1)$ is a factor of $f(x)$.

(ii) Hence or otherwise factorise $f(x)$ fully. **5**

(*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point. **1**

3. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

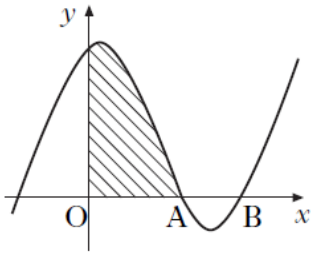
(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

4. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.



(*a*) Show that the graph cuts the x-axis at (3, 0). **1**

(*b*) Hence or otherwise find the coordinates of A. **3**

(*c*) Find the shaded area. **5**

5. (*a*) (i) Show that $(x-1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

(ii) Hence factorise $f(x)$ fully.

(*b*) Solve $2x^3 + x^2 - 8x + 5 = 0$.

(*c*) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

Find the coordinates of G.

(*d*) This tangent meets the curve again at the point H.

Write down the coordinates of H.

6. The roots of the equation $kx^2 - 3x + 2 = 0$ are equal.

What is the value of k?

A $-\frac{9}{8}$

B $-\frac{8}{9}$

C $\frac{8}{9}$

D $\frac{9}{8}$

7. Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p. **4**

8. Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots. **4**

9. Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

Logarithms and Exponentials

1. Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. **4**

2. Simplify $\dfrac{\log_b 9a^2}{\log_b 3a}$, where $a > 0$ and $b > 0$.

   A 2

   B $3a$

   C $\log_b 3a$

   D $\log_b(9a^2 - 3a)$

3. Solve the equation $\log_2(x + 1) - 2\log_2(3) = 3$. **4**

4. Functions f, g and h are defined on suitable domains by

   $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

   (a) Find expressions for $h(f(x))$ and $h(g(x))$. **3**

   (b) Hence solve $h(f(x)) - h(g(x)) = 3$. **5**

5. Solve the equation $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$. **4**

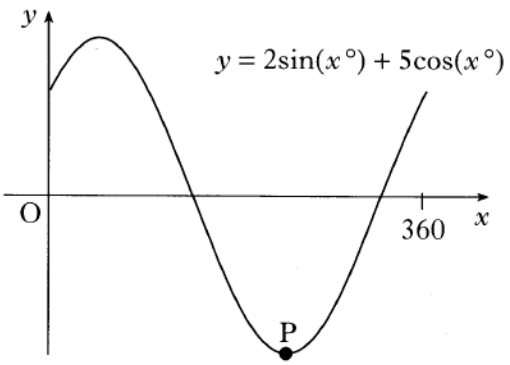
6. (a) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

   (ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

   (b) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

Wave Function and Double Angle

1. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.



(*a*) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \le a < 360$. **4**

(*b*) Find the coordinates of the minimum turning point P. **3**

2. (*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \le a \le 2\pi$. **4**

(*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \le x \le 2\pi$. **5**

3. (*a*) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \le x \le 2\pi$. **4**

4. (*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \le a \le 90$. **4**

(*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \le x \le 90$. **3**

5. If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$.

A $-\frac{3}{5}$

B $-\frac{2}{\sqrt{5}}$

C $\frac{2}{\sqrt{5}}$

D $\frac{3}{5}$

6. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$. **4**

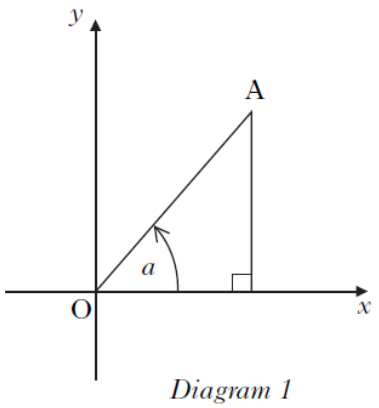


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

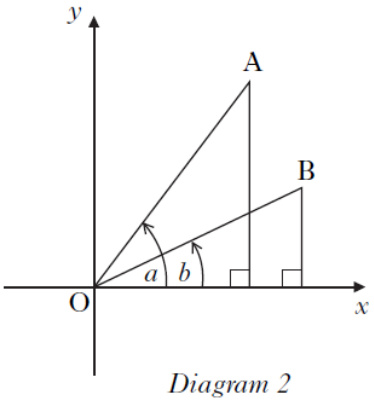
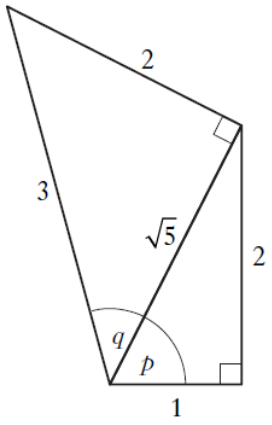


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$. **4**

7. The diagram shows two right-angled triangles with sides and angles as given.



What is the value of $\sin(p+q)$?

A $\frac{2}{\sqrt{5}}+\frac{2}{3}$

B $\frac{2}{\sqrt{5}}+\frac{\sqrt{5}}{3}$

C $\frac{2}{3}+\frac{2}{3\sqrt{5}}$

D $\frac{4}{3\sqrt{5}}+\frac{1}{3}$