8. INTEGRATION

8.1 BASIC INTEGRALS

(a) $\int 6x^2 - 5x \, dx = 2x^3 - \frac{5x^2}{2} + C$

(b) $\int 8 - 4x^{-\frac{1}{2}} \, dx = 8x - 8x^{\frac{1}{2}} + C$

$= 8x - 8\sqrt{x} + C$

(c) $\int_{-1}^{2} 3x - 4x^3 - 6 \, dx = \left[\frac{3x^2}{2} - x^4 - 6x \right]_{-1}^{2}$

$= \left(\frac{3(2)^2}{2} - (2)^4 - 6(2) \right) - \left(\frac{3(-1)^2}{2} - (-1)^4 - 6(-1) \right)$

$= -22 - \left(\frac{13}{2} \right)$

$= -\frac{57}{2}$

8.2 PRODUCTS AND QUOTIENTS

(a) $\int 10x - 5x^{\frac{3}{2}} \, dx = 5x^2 - 2x^{\frac{5}{2}} + C$

$= 5x^2 - 2\sqrt{x^5} + C$

(b) $\int \frac{x}{3x} - \frac{6x^2}{3x} \, dx = \int \frac{1}{3} - 2x \, dx$

$= \frac{1}{3}x - x^2 + C$

(c) $\int_{1}^{4} \frac{4x}{x^{\frac{1}{2}}} - \frac{3}{x^{\frac{1}{2}}} \, dx = \int_{1}^{4} 4x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \, dx$

$= \left[\frac{8x^{\frac{3}{2}}}{3} - 6x^{\frac{1}{2}} \right]_{1}^{4}$

$= \left[\frac{8\sqrt{x^3}}{3} - 6\sqrt{x} \right]_{1}^{4}$

$= \left(\frac{8\sqrt{4^3}}{3} - 6\sqrt{4} \right) - \left(\frac{8\sqrt{1^3}}{3} - 6\sqrt{1} \right)$

$= \frac{38}{3}$

8·3 DIFFERENTIAL EQUATIONS

(a) $\frac{dy}{dx} = 4x - 6x^2$

$y = 2x^2 - 2x^3 + c$

$(9) = 2(-1)^2 - 2(-1)^3 + c$

$9 = 2 + 2 + c$

$5 = c$

$\rightarrow \underline{\underline{y = 2x^2 - 2x^3 + 5}}$

(b) $f'(x) = 3x^2 - 6x + 1$

$f(x) = x^3 - 3x^2 + x + c$

$10 = (-2)^3 - 3(-2)^2 + (-2) + c$

$10 = -8 - 12 - 2 + c$

$32 = c$

$\underline{\underline{f(x) = x^3 - 3x^2 + x + 32}}$

8·4 FINDING AREAS

(a) $A = \int_0^2 (x^3 - 4x^2 + x + 6) - (0)\, dx$

$= \int_0^2 x^3 - 4x^2 + x + 6 \, dx$

$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x \right]_0^2$

$= \left(\frac{(2)^4}{4} - \frac{4(2)^3}{3} - \frac{(2)^2}{2} + 6(2) \right) - \left(\frac{(0)^4}{4} - \frac{4(0)^3}{3} + \frac{(0)^2}{2} + 6(0) \right)$

$= \underline{\underline{\frac{10}{3} \text{ unit}^2}}$

(b) $A = \int_0^4 (x^2 + 2x) - (x^3 - x^2 - 6x)\, dx$

$= \int_0^4 -x^3 + 2x^2 + 8x \, dx$

$= \left[-\frac{x^4}{4} + \frac{2x^3}{3} + 4x^2 \right]_0^4$

$= \left(-\frac{(4)^4}{4} + \frac{2(4)^3}{3} + 4(4)^2 \right) - \left(-\frac{(0)^4}{4} + \frac{2(0)^3}{3} + 4(0)^2 \right)$

$= \frac{272}{3}$ unit^2

8.5 UNKNOWN LIMITS

(a) $\int_1^p 2x - 5 \, dx = 18$

$\left[x^2 - 5x \right]_1^p = 18$

$(p^2 - 5p) - (1^2 - 5(1)) = 18$

$p^2 - 5p + 4 = 18$

$p^2 - 5p - 14 = 0$

$(p - 7)(p + 2) = 0$

$p = 7$ $\qquad$ ~~$p = -2$~~

$(\because p > 1)$

(b) $\int_0^k 6x^2 + 6x - 5 \, dx = 6$

$$\left[2x^3 + 3x^2 - 5x\right]_0^k = 6$$

$$\left(2k^3 + 3k^2 - 5k\right) - \left(2(0)^3 + 3(0)^2 - 5(0)\right) = 6$$

$$2k^3 + 3k^2 - 5k = 6$$

$$2k^3 + 3k^2 - 5k - 6 = 0$$

$$(k+1)(2k^2 + k - 6) = 0$$

$$(k+1)(2k-3)(k+2) = 0$$

$\cancel{k = -1}$ $\quad$ $\underline{\underline{k = \frac{3}{2}}}$ $\quad$ $\cancel{k = -2}$

$(\because k > 0)$

"try $k = 1$"

$$\begin{array}{r|rrrr} 1 & 2 & 3 & -5 & -6 \\ & & 2 & 5 & 0 \\ \hline & 2 & 5 & 0 & \boxed{-6} \end{array} \times$$

"try $k = -1$"

$$\begin{array}{r|rrrr} -1 & 2 & 3 & -5 & -6 \\ & & -2 & -1 & 6 \\ \hline & 2 & 1 & -6 & \boxed{0} \end{array} \checkmark$$

$\because$ rem $= 0$, $(k+1)$ is a factor.