

9. ADDITION FORMULAE

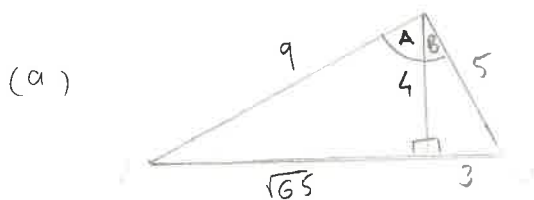
9.1 FINDING COMPOUND ANGLES

9.1.1 EXACT VALUES

$$\begin{aligned} \text{(a)} \quad \cos(105) &= \cos(60 + 45) \\ &= \cos 60 \cos 45 - \sin 60 \sin 45 \\ &= \left(\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \end{aligned}$$

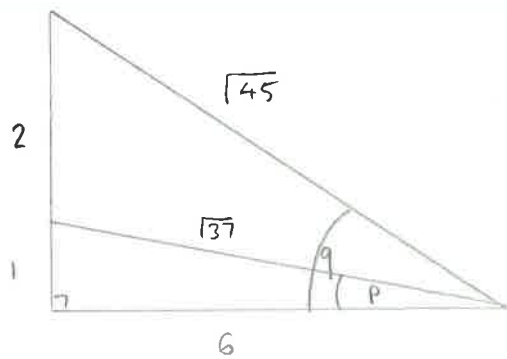
$$\begin{aligned} \text{(b)} \quad \sin(15) &= \sin(45 - 30) \\ &= \sin 45 \cos 30 - \cos 45 \sin 30 \\ &= \left(\frac{1}{\sqrt{2}}\right) \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \end{aligned}$$

9.1.2 TRIANGLE APPLICATIONS



$$\begin{aligned} \cos PQR &= \cos(A + B) \\ &= \cos A \cos B - \sin A \sin B \\ &= \left(\frac{4}{9}\right) \left(\frac{4}{5}\right) - \left(\frac{\sqrt{65}}{9}\right) \left(\frac{3}{5}\right) \\ &= \frac{16}{45} - \frac{3\sqrt{65}}{45} \\ &= \frac{16 - 3\sqrt{65}}{45} \end{aligned}$$

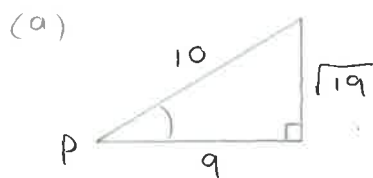
(b)



$$\begin{aligned}\sin(q - P) &= \sin q \cos P - \cos q \sin P \\ &= \left(\frac{3}{\sqrt{45}}\right) \left(\frac{6}{\sqrt{37}}\right) - \left(\frac{6}{\sqrt{45}}\right) \left(\frac{1}{\sqrt{37}}\right) \\ &= \frac{18}{\sqrt{45}\sqrt{37}} - \frac{6}{\sqrt{45}\sqrt{37}} \\ &= \frac{12}{\sqrt{45}\sqrt{37}} \quad \left(\frac{4}{\sqrt{185}}\right)\end{aligned}$$

9.2 FINDING DOUBLE ANGLES

9.2.1 TRIANGLE APPLICATIONS



$$\begin{aligned}\text{(i)} \quad \sin 2P &= 2 \sin P \cos P \\ &= 2 \left(\frac{\sqrt{19}}{10}\right) \left(\frac{9}{10}\right) \\ &= 2 \left(\frac{9\sqrt{19}}{100}\right) \\ &= \frac{9\sqrt{19}}{50}\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad \cos 2P &= 2 \cos^2 P - 1 \\ &= 2 \left(\frac{9}{10}\right)^2 - 1 \\ &= 2 \left(\frac{81}{100}\right) - 1 \\ &= \frac{31}{50}\end{aligned}$$

9.3 DOUBLE ANGLE EQUATIONS

$$(a) \quad 2 \sin x \cos x + \sin x = 0$$

$$\sin x (2 \cos x + 1) = 0$$

$$\downarrow$$

$$\sin x = 0$$



$$x = 0, 180, 360$$

$$\downarrow$$

$$\cos x = -\frac{1}{2}$$

$$RA = 60$$

✓ S	A
✓ T	C

$$x = 120, 240$$

$$\underline{\underline{x = 120, 180, 240, 360}}$$

$$(b) \quad 2 \cos^2 x - 1 - \cos x = 0$$

$$2 \cos^2 x - \cos x - 1 = 0$$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$\downarrow$$

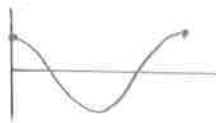
$$\cos x = -\frac{1}{2}$$

$$RA = 60$$

✓ S	A
✓ T	C

$$x = 120, 240$$

$$\cos x = 1$$



$$x = 0, 360$$

$$\underline{\underline{x = \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi}}$$

$$(c) \quad 3(2\cos^2 x - 1) = 1 - 10\cos x$$

$$6\cos^2 x - 3 = 1 - 10\cos x$$

$$6\cos^2 x + 10\cos x - 4 = 0$$

$$2(3\cos^2 x + 5\cos x - 2) = 0$$

$$2(3\cos x - 1)(\cos x + 2) = 0$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \cos x = \frac{1}{3} & \cdot & \cos x = -2 \end{array}$$

$$RA = 70.53^\circ$$

NO SOL^{NS} $\because -2 < -1$

S	A ✓
T	C ✓

$$\underline{\underline{x = 70.53^\circ, 289.47^\circ}}$$