Vectors

The Scalar Triple Product

The scalar triple product is a scalar **(not vector)** defined by:

$$\underline{a}.(\underline{b}\times\underline{c}) \quad \text{or} \quad \underline{a}.\underline{b}\times\underline{c}$$

Example

$\underline{a} = \underline{i} - \underline{j} + \underline{k}$, $\underline{b} = 2\underline{i} + 3\underline{j} + 4\underline{k}$, $\underline{c} = 3\underline{i} - 2\underline{j} + \underline{k}$

$$\underline{b}\times\underline{c} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 3 & 4 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= \underline{i}\begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} - \underline{j}\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} + \underline{k}\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$$

$$\underline{b}\times\underline{c} = \underline{i}\begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} - \underline{j}\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} + \underline{k}\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$$

$= (3 - (-8))\underline{i} - (2 - 12)\underline{j} + (-4 - 9)\underline{k}$

$= 11\underline{i} + 10\underline{j} - 13\underline{k}$

$$\underline{a}.\underline{b}\times\underline{c} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}.\begin{pmatrix} 11 \\ 10 \\ -13 \end{pmatrix}$$

$= 11 - 10 - 13$

$= -12$

Ex 5 Q 1, 3, 7

Or as a single calculation....

$$\underline{a}.(\underline{b}\times\underline{c}) \quad \text{or} \quad \underline{a}.\underline{b}\times\underline{c}$$

$\underline{a} = \underline{i} - \underline{j} + \underline{k}$, $\underline{b} = 2\underline{i} + 3\underline{j} + 4\underline{k}$, $\underline{c} = 3\underline{i} - 2\underline{j} + \underline{k}$

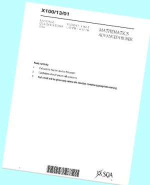
$$\underline{a}.(\underline{b}\times\underline{c}) = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 3 & 4 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= 1\times\begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} - (-1)\times\begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} + 1\times\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix}$$

$= 11 - 10 - 13$

$= -12$

2010 Q6 4 marks



The volume of a parallelepiped whose sides are given by the vectors **A**, **B**, and **C** is given by the absolute value of the scalar triple product

$$V_{\text{parallelepiped}} = |\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})|. \quad (10)$$

Parallelepiped

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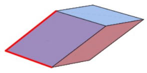

In three dimensions, a parallelepiped is a prism whose faces are all parallelograms. Let **A**, **B**, and **C** be the basis vectors defining a three-dimensional parallelepiped. Then the parallelepiped has volume given by the scalar triple product

$$V_{\text{parallelepiped}} = |\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})| \quad (1)$$
$$= |\mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})| \quad (2)$$
$$= |\mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})|. \quad (3)$$