Type 3: Q(x) the denominator has a linear factor and an irreducible quadratic factor.

Example 1

Express $\dfrac{3x^2-2x-5}{(x+2)(x^2-2x+3)}$ in partial fractions.

Firstly we must check that x^2-2x+3 does not factorise.

$a = 1$, $b = -2$, $c = 3$ and $b^2-4ac = -8$

Since $b^2-4ac<0$, no real roots exists $\leftrightarrow$ does not factorise.

$$\frac{3x^2-2x-5}{(x+2)(x^2-2x+3)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+3}$$

$$\frac{3x^2-2x-5}{(x+2)(x^2-2x+3)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-2x+3}$$

× both sides by $(x+2)(x^2-2x+3)$

$$3x^2-2x-5 = A(x^2-2x+3)+(Bx+C)(x+2)$$

Let $x = -2$ $\quad A = 1$

Let $x = 0$ $\quad C = -4$

Let $x = 1$ $\quad B = 2$

$$\frac{3x^2-2x-5}{(x+2)(x^2-2x+3)} = \frac{1}{x+2} + \frac{2x-4}{x^2-2x+3}$$

Example 2

Find partial fractions for $\dfrac{5x-7}{x^3+3x^2+2x+6}$.

Factorise denominator using synthetic division.

-3	1	3	2	6
		-3	0	-6
	1	0	2	0

$$\frac{5x-7}{x^3+3x^2+2x+6} = \frac{5x-7}{(x+3)(x^2+2)}$$

$$= \frac{A}{x+3} + \frac{Bx+C}{(x^2+2)}$$

$$\frac{5x-7}{x^3+3x^2+2x+6} = \frac{A}{x+3} + \frac{Bx+C}{\left(x^2+2\right)}$$

× both sides by $(x+3)(x^2+2)$

$$5x-7 = A(x^2+2) + (Bx+C)(x+3)$$

Let $x = -3$ $\quad A = -2$

Let $x = 0$ $\quad C = -1$

Let $x = 1$ $\quad B = 2$

$$\frac{5x-7}{x^3+3x^2+2x+6} = -\frac{2}{x+3} + \frac{2x-1}{x^2+2}$$