

# The Determinant

## The Determinant of a 2×2 matrix

The determinant of a square matrix is a number associated with the matrix.

Determinants have important applications and properties. The determinant of a square matrix  $A$  is denoted by  $\det(A)$  or  $|A|$ .

The determinant of a 2×2 matrix is defined as follows:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

If  $ad-bc = 0$ , there is no inverse and  $A$  is called **singular**.

### Example 1

$$\text{Find } \begin{vmatrix} 4 & 3 \\ -1 & -2 \end{vmatrix}$$

$$= -8 - (-3)$$

$$= \underline{\underline{-5}}$$

## The Determinant of a 3×3 matrix

The process for evaluating the determinant of a 3×3 matrix is best illustrated by example.

### Example 2

$$\text{Find } \det \begin{pmatrix} \overset{+}{3} & \overset{-}{2} & \overset{+}{-1} \\ 6 & -4 & 2 \\ -1 & 0 & 4 \end{pmatrix}$$

$$\begin{aligned} &= 3 \begin{vmatrix} -4 & 2 \\ 0 & 4 \end{vmatrix} - 2 \begin{vmatrix} 6 & 2 \\ -1 & 4 \end{vmatrix} + (-1) \begin{vmatrix} 6 & -4 \\ -1 & 0 \end{vmatrix} \\ &= 3(-16 - 0) - 2(24 - (-2)) + (-1)(0 - 4) \\ &= -96 \end{aligned}$$

### Example 3

$$\text{Find } \det \begin{pmatrix} \overset{+}{2} & \overset{-}{-3} & \overset{+}{4} \\ 1 & 2 & 1 \\ 0 & 5 & -2 \end{pmatrix}$$

$$= 2 \begin{vmatrix} 2 & 1 \\ 5 & -2 \end{vmatrix} - (-3) \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} + 4 \begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix}$$

$$= 2(-9) + 3(-2) + 4(5)$$

$$= \underline{\underline{-4}}$$

**Example 4**

Find the values of  $x$  for which  $\det \begin{pmatrix} x & 2 & 3 \\ 1 & x+1 & 4 \\ x+3 & 2 & 1 \end{pmatrix} = 31$

$$= x \begin{vmatrix} x+1 & 4 \\ 2 & 1 \end{vmatrix} - 2 \begin{vmatrix} 1 & 4 \\ x+3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 1 & x+1 \\ x+3 & 2 \end{vmatrix}$$

$$= x(x-7) - 2(1 - (4x+12)) + 3(2 - (x^2 + 4x + 3))$$

$$= x^2 - 7x + 8x + 22 - 3x^2 - 12x - 3$$

$$= -2x^2 - 11x + 19$$

$$\therefore -2x^2 - 11x + 19 = 31$$

$$-2x^2 - 11x - 12 = 0$$

$$2x^2 + 11x + 12 = 0$$

$$(2x + 3)(x + 4) = 0$$

*Sol<sup>n</sup> is  $x = -\frac{3}{2}$  and  $x = -4$*



**A useful rule.....**

(see 2007 Q5)

$$\det AB = \det A \times \det B$$