Complex Numbers

Polynomial Equations

It can be shown that:

- a polynomial equation of degree n has n roots (not necessarily all distinct or real)
- the complex roots of polynomial equations occur in conjugate pairs.

Example 1

Find the roots of $z^3 - 2z^2 - 8z + 21 = 0$

TESTING FOR ROOTS

f(1) ≠ 0 z - 1 is not a factor

f(-1) ≠ 0 z + 1 is not a factor

f(-3) = 0 z + 3 is a factor

USE SYNTHETIC DIVISION

$$\begin{array}{r|rrrr} -3 & 1 & -2 & -8 & 21 \\ & & -3 & 15 & -21 \\ \hline & 1 & -5 & 7 & 0 \end{array}$$

$$(z-3)(z^2-5z+7)$$

$$z = -3 \qquad z = \frac{5 \pm \sqrt{25-28}}{2}$$

$$= \frac{5 \pm \sqrt{-3}}{2}$$

$$= \frac{5 \pm \sqrt{3}i}{2}$$

So roots are $z = -3$, $\frac{5}{2} + \frac{\sqrt{3}}{2}i$, $\frac{5}{2} - \frac{\sqrt{3}}{2}i$

Example 2

$$f(z) = z^4 - 3z^3 + 5z^2 - 4z + 2$$

(a) show that $z = 1 + i$ is a root of $f(z) = 0$
(b) find all the roots of $f(z) = 0$

$1+i$	1	-3	5	-4	2
		$1+i$	$-3-i$	$3+i$	-2
	1	$-2+i$	$2-i$	$-1+i$	0

Hence $z = 1 + i$ is a root.

$z = 1 + i$ is a root => $z = 1 - i$ is also a root

$1-i$	1	$-2+i$	$2-i$	$-1+i$
		$1-i$	$-1+i$	$1-i$
	1	-1	1	0

$$z^2 - z + 1 = 0$$

$$z = \frac{1 \pm \sqrt{1-4}}{2}$$

$$= \frac{1 \pm \sqrt{3}i}{2}$$

=> roots are $z = 1 + i$, $1 - i$, $\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $\frac{1}{2} - \frac{\sqrt{3}}{2}i$