

Vectors

Angle between Two Planes

θ = angle between the normals of the planes.

The angle between two planes is the angle between their normal vectors.

Example

Find the size of the angle between the planes $x + 2y + z = 0$ and $x + y - z = 0$

Their normal vectors are $\underline{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$$\cos(\theta) = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}| |\underline{b}|} = \frac{1+2+0}{\sqrt{6}\sqrt{2}} = \frac{3}{\sqrt{12}} \Rightarrow \theta = 30^\circ$$

Example

For what values of α do the planes with equations

$$x + 2y - z = 1 \quad \text{and} \quad \alpha x - y + 3z = 2$$

intersect at right angles?

Their normals are $\underline{n}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\underline{n}_2 = \begin{pmatrix} \alpha \\ -1 \\ 3 \end{pmatrix}$, and we need $\underline{n}_1 \cdot \underline{n}_2 = 0$

So we require $1 \times \alpha + 2 \times (-1) + (-1) \times 3 = 0$

This gives $\alpha = 5$

ex 7A page 59 Q1, 2, 3

2013 Q15 3 marks
2008 Q14 3 marks

