Choosing a Suitable Substitution

In some simple cases, you will be required to decide on a suitable substitution yourself.

Integration by substitution is useful when the derivative of one part of the integrand is related to another part of the integrand.

Ex1 By choosing a suitable substitution $\int x\sqrt{x^2+1}\,dx$

Note that the derivative of $x^2 + 1$ is related to x.
This suggests that we should try the substitution $u = x^2 + 1$.

$$\int x\sqrt{x^2+1}\,dx$$

$$u = x^2 + 1$$

$$\tfrac{1}{2}du = xdx$$

$$\int x\sqrt{x^2+1}\,dx = \frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= \frac{1}{2}\times\frac{2}{3}\int u^{\frac{3}{2}} + C$$

$$= \frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}} + C$$

Ex 2 By choosing a suitable substitution $\int \frac{\sin x}{1+2\cos x}dx$

$$\int \frac{\sin x}{1+2\cos x}dx$$

$$u = 1 + 2cosxdx$$

$$du = -2sinxdx$$

$$-\tfrac{1}{2}du = sinxdx$$

$$\int \frac{\sin x}{1+2\cos x}dx = -\frac{1}{2}\int \frac{1}{u}du$$

$$= -\frac{1}{2}\ln\left|1+2\cos x\right| + C$$

2013

Q6 – 4 marks

Integrate $\dfrac{\sec^2 3x}{1+\tan 3x}$ with respect to x. **4**

$u = 1 + \tan 3x$ **OR** $u = \tan 3x$		•[1] Correct substitution.
$\dfrac{du}{dx} = 3\sec^2 3x$		•[2] Differentiates accurately
$\dfrac{1}{3}du = \sec^2 3x\,dx$		
$\int \dfrac{\frac{1}{3}du}{u}$ **OR** $\int \dfrac{\frac{1}{3}du}{1+u} = \cdots$		•[3] Correct substitution of d and $f(u)$ into integral.
$= \dfrac{1}{3}\ln\lvert u\rvert + c$ **OR** $= \dfrac{1}{3}\ln\lvert 1+u\rvert + c$		
$= \dfrac{1}{3}\ln\lvert 1+\tan 3x\rvert + c$		•[4] Integrates correctly *and* substitutes back.[1,2,3]

2011

Q11 – 7 marks

(*a*) Obtain the exact value of $\int_0^{\pi/4} (\sec x - x)(\sec x + x)\,dx$. **3**

(*b*) Find $\int \dfrac{x}{\sqrt{1-49x^4}}\,dx$. **4**

(b) *Method 1*		
Let $u = 7x^2$,	**M1**	
then $du = 14x\,dx$.	**1**	
$\int \dfrac{x}{\sqrt{1-49x^4}}\,dx = \dfrac{1}{14}\int \dfrac{du}{\sqrt{1-u^2}}$	**1**	
$= \dfrac{1}{14}\sin^{-1} u + c$		
$= \dfrac{1}{14}\sin^{-1} 7x^2 + c$	**1**	must be in terms of x