Binomial

2013

Q1 – 4 marks

Write down the binomial expansion of $\left(3x - \frac{2}{x^2}\right)^4$ and simplify your answer.

Marking Instructions

Expected Answer/s	Max Mark	Additional Guidance
Write down the binomial expansion of $\left(3x - \frac{2}{x^2}\right)^4$ **and simplify your answer.**	4	
${}^4C_0(3x)^4\left(\frac{-2}{x^2}\right)^0 + {}^4C_1(3x)^3\left(\frac{-2}{x^2}\right)^1 + {}^4C_2(3x)^2\left(\frac{-2}{x^2}\right)^2$ $+ {}^4C_3(3x)^1\left(\frac{-2}{x^2}\right)^3 + {}^4C_4(3x)^0\left(\frac{-2}{x^2}\right)^4$		•[1] Correct binomial coefficients.[2] •[2] Correct powers of $3x$ and $\frac{-2}{x^2}$.
$= 81x^4 + 4\cdot 27x^3 \cdot \frac{-2}{x^2} + 6\cdot 9x^2 \cdot \frac{4}{x^4} + 4\cdot 3x \cdot \frac{-8}{x^6} + \frac{16}{x^8}$		•[3] Simplifies indices.[1] •[4] Completes simplification of coefficients.[3]
$= 81x^4 - 216x + \frac{216}{x^2} - \frac{96}{x^5} + \frac{16}{x^8}$		

Notes:
1.1 Accept negative indices.
1.2 Award •[1] nCr or $\binom{n}{r}$ form.
1.3 Including signs. " + -" or "- -" : do not award •[4]
1.4 Expanding wrong expression: $\left(3x - \frac{2}{x}\right)^4$, •[1] •[4] only are available.
1.5 Expanding $\left(3x + \frac{2}{x^2}\right)^4$, •[1] •[3] •[4] only are available.

2012

Q4 – 5 marks

Write down and simplify the general term in the expansion of $\left(2x - \frac{1}{x^2}\right)^9$. 3
Hence, or otherwise, obtain the term independent of x. 2

Marking Instructions

The general term is given by:

$$\binom{9}{r}(2x)^{9-r}\left(-\frac{1}{x^2}\right)^r \quad 1$$

$$= \binom{9}{r} \times \frac{2^{9-r}x^{9-r}(-1)^r}{x^{2r}} \quad 1$$

$$= \binom{9}{r} \times (-1)^r 2^{9-r} x^{9-3r} \quad 1$$

The term independent of x occurs when
$9 - 3r = 0$, i.e. when $r = 3$. 1

The term is: $\frac{9!}{6!\,3!}(-1)^3 2^6$
$= -5376.$ 1

2011

Q2 – 3 marks

Use the binomial theorem to expand $\left(\frac{1}{2}x-3\right)^4$ and simplify your answer.

Marking Instructions

$\left(\frac{1}{2}x-3\right)^4 = {}^4C_0\left(\frac{x}{2}\right)^4 + {}^4C_1\left(\frac{x}{2}\right)^3(-3) + {}^4C_2\left(\frac{x}{2}\right)^2(-3)^2 + {}^4C_3\left(\frac{x}{2}\right)(-3)^3 + {}^4C_4(-3)^4$	1 1	for powers for coefficients
$= \left(\frac{x}{2}\right)^4 + 4\left(\frac{x}{2}\right)^3(-3) + 6\left(\frac{x}{2}\right)^2(-3)^2 + 4\left(\frac{x}{2}\right)(-3)^3 + (-3)^4$		
$= \frac{x^4}{16} - \frac{3x^3}{2} + \frac{27x^2}{2} - 54x + 81.$	1	for simplifying

2010

Q5 – 4 marks

Show that

$$\binom{n+1}{3} - \binom{n}{3} = \binom{n}{2}$$

where the integer n is greater than or equal to 3.

Marking Instructions

$\binom{n+1}{3} - \binom{n}{3} = \frac{(n+1)!}{3!(n-2)!} - \frac{n!}{3!(n-3)!}$	1	both terms correct
$= \frac{(n+1)!}{3!(n-2)!} - \frac{n!(n-2)}{3!(n-2)!}$		{alternative methods will appear}
$= \frac{(n+1)! - n!(n-2)}{3!(n-2)!}$		
$= \frac{n![(n+1)-(n-2)]}{3!(n-2)!}$	1 1	correct numerator correct denominator
$= \frac{n! \times 3}{3!(n-2)!} = \frac{n!}{2!(n-2)!}$	1	
$= \binom{n}{2}$		1 for knowing (anywhere) $(n-2)! = (n-2)\times(n-3)!$

2009

Q8 – 3 marks

(*a*) Write down the binomial expansion of $(1 + x)^5$.

(*b*) Hence show that $0{\cdot}9^5$ is $0{\cdot}59049$.

Marking Instructions

(a) $(1 + x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$ **1**

(b) Let $x = -0{\cdot}1$, then **1**

$0{\cdot}9^5 = (1 + (-0{\cdot}1))^5$

$= 1 - 0{\cdot}5 + 0{\cdot}1 - 0{\cdot}01 + 0{\cdot}0005 - 0{\cdot}00001$ **1**

$= 0{\cdot}5 + 0{\cdot}09 + 0{\cdot}00049$

$= 0{\cdot}59049$

2008

Q8 – 5 marks

Write down and simplify the general term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{10}$. **3**

Hence, or otherwise, obtain the term in x^{14}. **2**

Marking Instructions

The rth term is

$\binom{10}{r}(x^2)^{10-r}\left(\frac{1}{x}\right)^r$ **1 for form**
1 for powers

$= \binom{10}{r}x^{20-3r}$ **1 for simplifying**

$20 - 3r = 14 \Rightarrow r = 2$ **1**

term is $45x^{14}$ **1**

or

$\binom{10}{r}(x^2)^r\left(\frac{1}{x}\right)^{10-r} = \binom{10}{r}x^{3r-10}$ **1,1,1**

$3r - 10 = 14 \Rightarrow r = 8$ **1**

term is $45x^{14}$ **1**

2007

Q1 – 4 marks

Express the binomial expansion of $\left(x - \frac{2}{x}\right)^4$ in the form $ax^4 + bx^2 + c + \frac{d}{x^2} + \frac{e}{x^4}$ for integers a, b, c, d and e.

Marking Instructions

$$\left(x - \frac{2}{x}\right)^4 = x^4 + 4x^3\left(-\frac{2}{x}\right) + 6x^2\left(-\frac{2}{x}\right)^2 + 4x\left(-\frac{2}{x}\right)^3 + \left(-\frac{2}{x}\right)^4$$ **1 for powers**
1 for coeffs

$$= x^4 - 8x^2 + 24 - \frac{32}{x^2} + \frac{16}{x^4}$$ **2E1**

2005

Q12 (a) – 3 marks *also involves "Complex Numbers" from Unit 3*

Let $z = \cos\theta + i\sin\theta$.

(*a*) Use the binomial expansion to express z^4 in the form $u + iv$, where u and v are expressions involving $\sin\theta$ and $\cos\theta$.

Marking Instructions

(a) $z^4 = (\cos\theta + i\sin\theta)^4$

$= \cos^4\theta + 4\cos^3\theta(i\sin\theta) + 6\cos^2\theta(i^2\sin^2\theta) + 4\cos\theta(i^3\sin^3\theta) + i^4\sin^4\theta$ **M1**

$= \cos^4\theta + 4i\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta - 4i\cos\theta\sin^3\theta + \sin^4\theta$ **1**

$= (\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta) + i(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta)$ **1**

2004

Q2 – 3 marks

Obtain the binomial expansion of $(a^2 - 3)^4$.

Marking Instructions

$(a^2 - 3)^4 = (a^2)^4 + 4(a^2)^3(-3) + 6(a^2)^2(-3)^2 + 4(a^2)(-3)^3 + (-3)^4$

1 for binomial coefficients

$= a^8 - 12a^6 + 54a^4 - 108a^2 + 81$ **1 for powers**

1 for coefficients

2003

A9 (a bit of) – 2 marks

Expand $(w + w^{-1})^4$ by the binomial theorem

Marking Instructions

$(w + w^{-1})^4 = w^4 + 4w^2 + 6 + 4w^{-2} + w^{-4}$ **2E1**