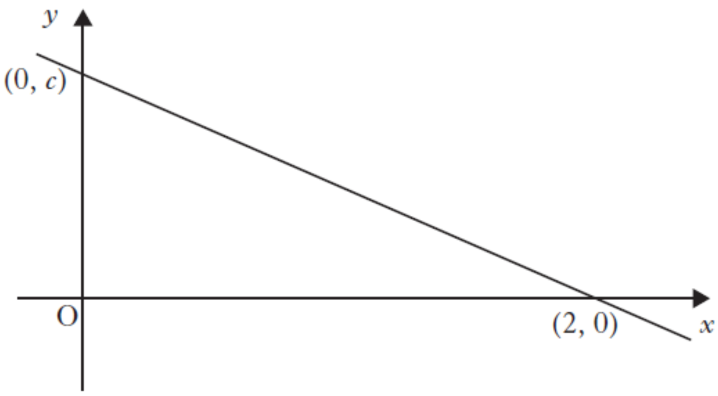
Curve Sketching

2013

Q13 – 5 marks

Part of the straight line graph of a function $f(x)$ is shown.

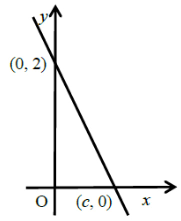
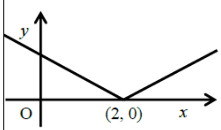


(a) Sketch the graph of $f^{-1}(x)$, showing points of intersection with the axes. **2**

(b) State the value of k for which $f(x) + k$ is an odd function. **1**

(c) Find the value of h for which $|f(x + h)|$ is an even function. **2**

Marking Instructions

a			$\bullet^1$ Straight line with negative gradient crossing the positive sections of the x- and y-axes. $\bullet^2$ *Both* intersections correctly annotated.
b	$y = f(x) - c$ is odd. $\therefore \boldsymbol{k = -c}$		$\bullet^3$ Correctly stated.
c	 $y = \lvert f(x+2) \rvert$ is even $\therefore \boldsymbol{h = 2}$		$\bullet^4$ Sketch of $y = \lvert f(x) \rvert$ with point of reflection marked. $\bullet^5$ Explicit statement of answer.

Notes:

13.1 Answer $h = 2$ only, no other working or diagram, award full marks [2 out of 2].
13.2 Where a candidate has clearly used their diagram from part (a) as the basis for (b) and (c), leading to $k = -2$ and $h = c$ (with working/further diagram) award $\bullet^4$ $\bullet^5$ and not $\bullet^3$ (2 out of the three marks for (a) and (b)). Statement of above answers only, zero out of 3.
13.3 An accurate diagram of $y = f(x + 2)$, on its own, gains no marks.

2012

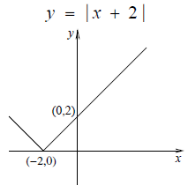
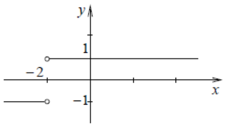
Q7 – marks

A function is defined by $f(x) = |x + 2|$ for all x.

(*a*) Sketch the graph of the function for $-3 \le x \le 3$. **2**

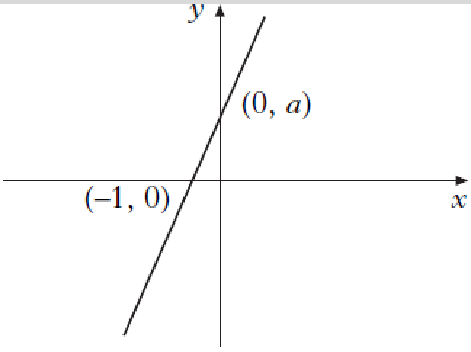
(*b*) On a separate diagram, sketch the graph of $f'(x)$. **2**

Marking Instructions

(a)		1 1	for shape for coordinates
(b)		1 1	for both horizontal lines for values: 1, −1, −2

2011

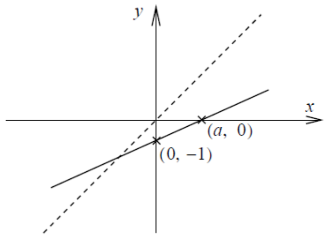
Q6 – 4 marks



The diagram shows part of the graph of a function $f(x)$. Sketch the graph of $|f^{-1}(x)|$ showing the points of intersection with the axes. **4**

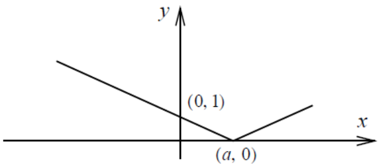
Marking Instructions

Reflect in the line $y = x$ to get



1 for position
1 for coordinates

Now apply the modulus function

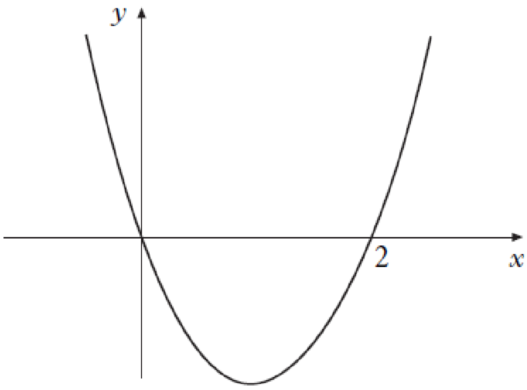


1 for shape
1 for coordinates

2010

Q10 – 3 marks

The diagram below shows part of the graph of a function $f(x)$. State whether $f(x)$ is odd, even or neither. Fully justify your answer.



3

Marking Instructions

The graph is not symmetrical about the y-axis (or $f(x) \neq f(-x)$) so it is not an even function.	1	
The graph does not have half-turn rotational symmetry (or $f(x) \neq -f(-x)$) so it is not an odd function.	1	
The function is neither even nor odd.	1	*{apply follow through}*

2009

Q13 – 10 marks

The function $f(x)$ is defined by

$$f(x) = \frac{x^2 + 2x}{x^2 - 1} \qquad (x \neq \pm 1).$$

Obtain equations for the asymptotes of the graph of $f(x)$. **3**

Show that $f(x)$ is a strictly decreasing function. **3**

Find the coordinates of the points where the graph of $f(x)$ crosses

(i) the x-axis and

(ii) the horizontal asymptote. **2**

Sketch the graph of $f(x)$, showing clearly all relevant features. **2**

Marking Instructions

$$f(x) = \frac{x^2 + 2x}{x^2 - 1} = \frac{x^2 + 2x}{(x - 1)(x + 1)}$$

Hence there are vertical asymptotes at $x = -1$ and $x = 1$. **1**

$$f(x) = \frac{x^2 + 2x}{x^2 - 1} = \frac{1 + \frac{2x}{x^2}}{1 - \frac{1}{x^2}} = \frac{1 + \frac{2}{x}}{1 - \frac{1}{x^2}}$$ **1**

$$\to 1 \text{ as } x \to \infty.$$

So $y = 1$ is a horizontal asymptote. **1**

$$f(x) = \frac{x^2 + 2x}{x^2 - 1}$$

$$f'(x) = \frac{(2x + 2)(x^2 - 1) - (x^2 + 2x)2x}{(x^2 - 1)^2}$$ **1**

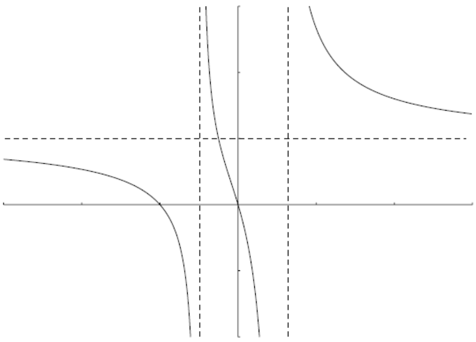
$$= \frac{2x^3 - 2x + 2x^2 - 2 - 2x^3 - 4x^2}{(x^2 - 1)^2} = \frac{-2(x^2 + x + 1)}{(x^2 - 1)^2}$$ **1**

$$= \frac{-2\left((x + \frac{1}{2})^2 + \frac{3}{4}\right)}{(x^2 - 1)^2} < 0$$ **1**

Hence $f(x)$ is a strictly decreasing function.

$$f(x) = \frac{x^2 + 2x}{x^2 - 1} = 0 \Rightarrow x = 0 \text{ or } x = -2$$ **1**

$$f(x) = \frac{x^2 + 2x}{x^2 - 1} = 1 \Rightarrow x^2 + 2x = x^2 - 1 \Rightarrow x = -\frac{1}{2}$$ **1**



1 for shape
1 for position

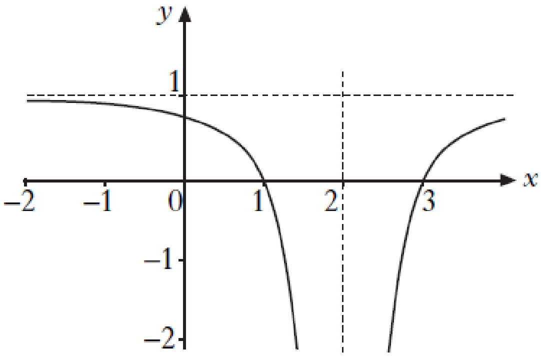
Alternatively for the horizontal asymptote:

$$\begin{array}{r|l} & \quad\quad\quad 1 \\ x^2 - 1 & x^2 + 2x \\ & x^2 \quad\;\; - 1 \\ \hline & \quad 2x + 1 \end{array} \Rightarrow f(x) = 1 + \frac{2x + 1}{x^2 - 1} \to 1 \text{ as } x \to \infty$$

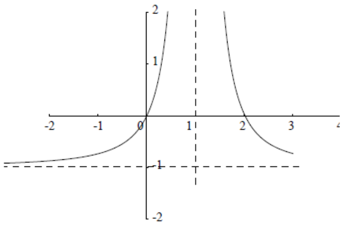
2008

Q3 – 4 marks

Part of the graph $y = f(x)$ is shown below, where the dotted lines indicate asymptotes. Sketch the graph $y = -f(x+1)$ showing its asymptotes. Write down the equations of the asymptotes. **4**



Marking Instructions



1 for inverting

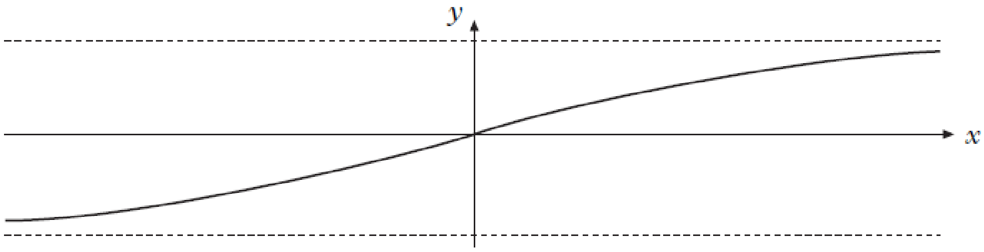
1 for translation

1 for showing asymptotes

Asymptotes are $y = -1$ and $x = 1$. **1**

2007

Q16 a & c – marks



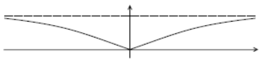
(*a*) The diagram shows part of the graph of $f(x) = \tan^{-1} 2x$ and its asymptotes. State the equations of these asymptotes. **2**

(*c*) Sketch the graph of $y = |f(x)|$ and calculate the area between this graph, the x-axis and the lines $x = -\frac{1}{2}$, $x = \frac{1}{2}$. **3**

Marking Instructions

(a) $\tan^{-1} 2x$ has horizontal asymptotes at $y = \pm\frac{\pi}{2}$. **1,1**

(c)



2E1

$$\int_{-1/2}^{1/2} |f(x)|\,dx = 2\int_0^{1/2} \tan^{-1} 2x\,dx$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln 2$$ **1**